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## Measurement of shutdown margin

Knud Borge Pedersen  
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MEASUREMENT OF SHUTDOWN MARGIN.

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MEASUREMENT OF SHUTDOWN MARGIN

by

Knud Borge Pedersen

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## INTRODUCTION

In the operation of a nuclear reactor it is often desirable to know what amount of reactivity is present at any given time. This is important from the standpoint of optimum safety in operation, since the criticality of a reactor is determined by the amount of reactivity.

With the trend towards larger reactors, some of which use the module concept, it would be advantageous to be able to measure the reactivity in several regions of the reactor. In the operation of such large reactors the possibility exists that a region is slightly subcritical, due to the insertion of too much absorber, although the reactor as a whole is critical. To operate in this manner would be uneconomical because of uneven burnup, and might be prevented if the reactivity had been known for several regions of the reactor.

A very useful method by which the reactivity and other reactor parameters may be obtained is analysis of the frequency dependent reactor transfer function. The measurement of the reactor transfer function at some point in the reactor does not depend on a theoretical description of the system, since the transfer function is only a measure of the reactor's response to some input disturbance.

It was the purpose of this investigation to explore a method whereby the reactor transfer function measured in

a region of a reactor could be used to measure directly its negative reactivity.



## REVIEW OF THE LITERATURE

It is possible to determine the instantaneous state (i.e. the amount of criticality) of a reactor by a number of different calculations if the various physical parameters of the reactor are available for the calculations at that moment. Since the parameters change with the state of the reactor this is obviously a very difficult, if not impossible task. Several methods have been suggested for measuring the state, but most are only practical when the reactor is critical or supercritical. The method which is used in reactor operations depends on measuring the reactor period, but it only gives an approximate indication of the state.

J. M. Harrer at Argonne National Laboratory is believed to be the first to suggest that a nuclear reactor may be regarded as a circuit element and that it can be described by its transfer function. An experiment was carried out on the CP-2 reactor at Argonne and a 1952 article by J. M. Harrer, R. E. Boyar, and Darwin Krucoff (10) described the experiment. The flux in the reactor was caused to oscillate by placing a cadmium cylinder-piston arrangement near the center of the reactor. The piston was moved in the cylinder in a sinusoidal manner by a cam arrangement, and an ion chamber was placed in the reactor to detect the change in the neutron flux. The amplitude and phase shift of the theoretical transfer function were calculated as functions

of frequency, using five groups of delayed neutrons. The experimental values of the transfer function were determined, and the amplitude, after being normalized to the amplitude of the theoretical transfer function at the break frequency, was found to agree closely with the experimental amplitude for all the investigated frequencies. The experimental phase shift was corrected for the phase shift introduced by the ion chamber-amplifier circuit, which was a function of frequency, and was then found to correspond to the theoretically determined phase shift at all frequencies. It had thus been shown experimentally that the transfer function of a reactor is a measurable quantity.

The method employed by Harrer et al. was later refined by including a "nulling" system. This system is described by A. H. Wasserman (24) and is claimed to be capable of measuring the phase angle with an accuracy of 0.5% and the magnitude with an accuracy of 1%. This is achieved by passing the current from the ion chamber through the series combination of a sine potentiometer and an adjustable resistance. The sine pot is driven synchronously with the oscillator and its phase can be adjusted by a differential gear. The voltage at the wiper of the sine pot is passed through a bandpass filter which is tuned to the oscillator frequency. Two ac components appear which by turning the pot can be made of opposite phase. A null output can be achieved by adjustment

of the resistor. The phase of the signal is simply related to the phase of the sine pot, and the amplitude of the ac signal is simply related to the resistance. Wasserman stated that the system could be used for low power tests but not for high power tests. It would also appear that it cannot be used for subcritical operation with a sufficient degree of accuracy due to the small signal to noise ratio. The system also requires some subsequent calculations to be made on the data because the electrical circuit is changed for every reading so that an electrical correction must be made.

The reactor transfer function may be used to find the state of a reactor. Uhrig (22) suggests the method of noise analysis as a measure of the reactor shutdown margin and further suggests that this may be accomplished by both power-spectral-density and cross-correlation techniques. Both of these are based on statistical treatment of the signal received from a detector. The first method uses the autocorrelation function to find the power spectral density of the output signal using a relationship which says that the power spectral density of the output equals the product of the square of the magnitude of the transfer function of the reactor and the power spectral density of the input,

$\bar{\Phi}_{oo}(\omega) = |G(j\omega)|^2 \bar{\Phi}_{ii}(\omega)$  (4). The latter may usually be considered constant, so that it is only necessary to measure the output signal and compare it with the output signal at

some previous time to obtain the autocorrelation function; a Fourier transformation then leads to the power spectral density of the output,  $\Phi_{00}(\omega)$ . By doing this over a wide range of frequencies, the transfer function can be found by fitting the data to the curve given by the theoretical expression. The cross-correlation technique suggested by Uhrig depends on using an input in the form of a modulated neutron beam which is correlated with the output signal.

The advantage of the first method is that it uses only the output of the system itself for the determination; however, the method gives no information about phase angle, and background noise has serious detrimental effects. It is also obvious that considerable time is required to obtain the transfer function at any state of the reactor. The cross-correlation technique has the advantage of being insensitive to noise but the method, as suggested by Uhrig, requires a neutron generator.

Stern and Valat (21) used a method similar to the one suggested by Uhrig to measure experimentally the reactivity in the far subcritical range. A digital counting system was used to cross-correlate the output with the input, the input being in the form of a neutron wave which was modulated by a pseudorandom square wave. Their paper presented only a tentative set of results due to the fact that insufficient equipment was available. Their conclusions were that the method was feasible, but it does require a large number

of scalers.

Albrecht (2) discussed a system for accurately measuring the reactor noise, which is an essential part of any of the procedures noted. He analyzed the noise over a range of frequencies from 0.01 cycles/sec to about 1 kc and detected relative amplitudes varying over a range of  $10^3$ . The procedure involved the tape-recording of the signal emitted by a detector located near the reactor core. One advantage gained by introducing the additional step of taping the information is that reactor drift which may take place over the relatively long time needed for analysis can be eliminated by taping over a short time interval, making a loop from the tape, and analyzing the tape loop over a wide range of frequencies. Another advantage is that the tape can be replayed at higher speeds than those at which it was recorded to make it possible for the analyzer to handle frequencies below 10 cycles/sec.

Schultz (19) used the power-spectral-density measurement to find the transfer function of a reactor, however, he introduced a new method for finding the negative reactivity. From the plot of the transfer function amplitude as a function of frequency he saw that the ratio of the amplitudes at two fixed frequencies is a function of the shutdown reactivity. Thus by measuring the amplitude of the transfer function at eg. 10 and 600 cps, and calculating the ratio  $A(600)/A(10)$

he determined an experimental curve which, although it differed from the theoretical curve because of uncorrelated noise, gave a smooth function of the shutdown reactivity. It had the additional advantage that the ratio of the two amplitudes became constant when the reactor was critical. He was able to determine this ratio down to a shutdown reactivity of  $-8\% \Delta k/k$  in a cold, clean critical assembly. He pointed out, however, that generally the power spectral density measurements for large values of subcriticality are obscured by bombardment noise and by extraneous instrument noise, and in high-power reactors by shutdown gamma noise.

A program was set up at Oak Ridge National Laboratory by Ricker et al. (16) to establish a theoretical and experimental basis for neutron fluctuation measurements. Their first goal was to develop a continuous method of measuring the negative reactivity by neutron fluctuation analysis. Their concept was similar to that of Schultz but used the spectral densities of six fixed-frequency intervals in the high-frequency portion of the neutron fluctuation spectrum; frequency intervals were selected above and below the upper break frequency of the reactor transfer function, and the ratio of the spectral densities of two frequency intervals was used as a measure of the reactivity. The main difference between their method and that of Schultz was in the determination of the output spectrum density which was

here determined over a frequency interval rather than at a fixed frequency. The spectral density in a fixed frequency interval  $\Delta\omega$  is related to the mean square of the amplitude of the fluctuation  $x(t)$  from the detector, and is given by the expression  $\bar{\Phi}_{00}(\omega_c) = \frac{\overline{x(t)^2}}{\Delta\omega}$ , where  $\omega_c$  is the center frequency of the interval  $\Delta\omega$  and  $\bar{\Phi}_{00}(\omega)$  is assumed constant through  $\Delta\omega$ . The ratio  $R = \bar{\Phi}_{00}(\omega_1)/\bar{\Phi}_{00}(\omega_2)$  was then a measure of the reactivity with  $\omega_1$  and  $\omega_2$  selected for sensitivity of  $R$  to reactivity. The results presented in the paper were of a preliminary nature, but it was stated that approximately 80% of the experimental data points were within 30% of the reference reactivities. Measurements were made to a negative reactivity of approximately  $1\beta$ ; if  $\beta$  is taken as 0.007 this would equal a  $\Delta k$  of -0.007.

A later paper by Valat (23) explained a method of extracting data from an experimental curve obscured by noise. Instead of using the method of least squares to fit the experimental curve of input-output cross correlation for a given random input excitation to an analytical form of the curve, he suggested that it is possible not to take into account the actual errors, but to eliminate them by considering them as noise. After the true experimental curve has thus been obtained by tightening the variance around the curve, the analytical form is introduced later. He called this method the autocorrelation of the cross correlation. He

found that the method gave smaller relative statistical errors than the method of least squares for pulses which are Poisson distributed.

Bryce (6) developed the time-dependent probability for detection of a prompt thermal neutron following the detection of a prompt fission neutron at some arbitrary origin of time. He then showed that it is possible to measure reactivity and power through the measurement of this probability. He considered an all-thermal heterogeneous point reactor with one group of delayed neutrons. The prompt neutron production can then be described as  $dN/dt = -\alpha N$  which leads to the well-known exponential that describes the time behavior of the prompt neutron population in a multiplying medium,  $N = N_0 e^{-\alpha t}$ . He then considered that any detector counts are concurrent with a corresponding fission, and used the time at which the first detection occurred as the reference time.  $\nu$  neutrons are released per fission; if one of the  $\nu$  neutrons is absorbed in the detector, the neutron population in the reactor will change as  $N = \nu e^{-\alpha t}$ . If  $\Lambda$  is the mean time for production of a neutron,  $\nu\Lambda$  is the mean time for a fission event, and the probability per unit time that a single neutron will cause fission is  $(\nu\Lambda)^{-1}$ . It then follows that since we have  $N$  neutrons the probability per unit time that a subsequent fission will occur is  $\frac{N}{\nu\Lambda} = \nu e^{-\alpha t} / \nu\Lambda$ . The probability that a fission is detected is then obtained by multiplying



the expression by the detector efficiency  $E$ , which is the number of counts per fission. Since many fissions are taking place at the same time there will also be a background count rate,  $R$ , superimposed on the probability per unit time that a detector count is obtained at time  $t$ , thus the probability per unit time that a detector count is obtained at  $t$  following a count at  $t = 0$  is  $P(t) = Ee^{-\alpha t}/\Lambda + R$ .

The experiment was performed by inserting a detector into the reactor and recording the count rate with a multi-channel analyzer operating in the time mode. One of the counts from the detector was used to trigger the analyzer. Thirty thousand analysis cycles, each of 3200  $\mu$  sec duration, were used, resulting in 800 counts in the first channel. From the relationship for  $P(t)$  and the experimental data,  $\alpha$  and the reactivity was found. The shutdown margin was found for all control elements fully inserted to be  $\$12.73 \pm 0.50$ .

A different approach was suggested by Moore (12), from whose method came what appeared to be a new criterion for criticality. Moore showed that the state function of a reactor can be expressed as a linear superposition of normal modes of the form  $\psi(\bar{q}, t) = \sum_i \bar{\Psi}_i(k_i, t) \bar{\Phi}_i(k_i, \bar{q})$ , where  $\bar{q}$  is a general position vector, and where the spatial functions  $\bar{\Phi}_i$  are three-dimensional eigenfunctions of the Laplacian corresponding to the eigenvalue  $-k_i^2$ , and where  $\bar{\Psi}_i$  is the time dependent solution. He particularized to the case of age-diffusion theory in cylindrical coordinates. The time

dependent waves will be characterized by their frequencies, and his calculations showed that there is a connection between criticality and the existence of pass frequencies, which he defined as frequencies that permit no attenuation of the neutron waves through the reactor; he also defined stop frequencies as those frequencies which permit no harmonic dependence upon distance travelled through the reactor. The two types of frequency were called exceptional frequencies.

A paper by Brehm (5) shows that for thermal reactor systems characterized by a thermal diffusion lifetime at least as large as the slowing down time, all non-zero exceptional frequencies are stop frequencies. Moore's approach depended on the existence of pass frequencies and since, according to Brehm, the pass frequencies for such a system would have a frequency of zero, there is a practical difficulty with the method of how to measure or generate neutron waves of zero frequency.

A reactivity meter was built for the SRE and was reported by Pellet (14b). The meter was installed to provide an immediate indication of abnormalcy by continuously computing reactivity changes due to temperature, power, and poisoning effects. The meter actually measured the effect of each of the above changes; by using the initial condition of the reactor as a reference condition and adding the reactivity caused by the changes, the predicted reactivity was obtained.

Thus the predicted reactivity,  $\rho_p = \rho_B + \rho_{P\&T} + \rho_{Xe} + \rho_{Sm} + \rho_{Bu}$ , where  $\rho_B$  is the excess reactivity at zero power which is built into the reactor, and the other terms are the reactivities due to power and temperature, xenon poison, samarium poison, and burn-up respectively.

Simple linear relationships were set up describing each of the reactivities in terms of constants and one measured quantity. These quantities were the temperature, which was measured by thermocouples, and xenon concentration which was computed by an analog computer with measured flux and time as input variables. The meter as it was constructed used hand calculations for the last two reactivity terms because of their long term predictability. The report showed a plot of measured and calculated reactivity, and there seems to be fairly good agreement. It should be pointed out that it is not a true reactivity meter because it is only indirectly using information from the reactor. Since a measurement is based on the built-in reactivity it seems that effects omitted from the calculations being performed by the computers during operation will cause the measured reactivity to increasingly deviate from the actual reactivity. The meter also will not measure the shutdown margin.

The most recent contributions to the field of reactivity measurement by noise analysis were made independently by Seifritz et al. (20) in Germany and by Nomura et al. (13) in

Japan. In both cases the reactors were externally unperturbed, and two detectors placed side by side were used for the measurements. The outputs from the two detectors were cross-correlated, thus suppressing the uncorrelated noise in the signal, and allowing a much smaller ratio of correlated to uncorrelated signal to yield reliable results. Seifritz et al. report that they were able to go from a signal to noise ratio of 2 to a ratio of only 0.1 by going to the two-detector method and increasing the analysis time.

An article by Corngold (7) points out a difficulty which is shared by all who do research on subcritical reactivity. The difficulty exists because no agreement has been reached on nomenclature. Corngold shows that one may define reactivity by one expression for a bare, homogeneous slab reactor, using one-velocity diffusion theory, and that the same expression will not necessarily hold when multi-velocity and/or transport effects are present.

In order to avoid any confusion this paper will refer to the multiplication factor rather than the reactivity.

## OBJECTIVE AND DESCRIPTION OF THE PROBLEM

The objective of this investigation was to measure the reactivity of a nuclear reactor operating in a steady state below the critical condition.

The method which was used to accomplish this was to induce a sinusoidal neutron wave in the reactor by means of an oscillator. The oscillator consisted of a sine shaped absorber rotating past a fixed absorber. The measured quantity was the amplitude of the cross correlation of a signal proportional to the instantaneous neutron flux and the signal from a sine-generating potentiometer which was attached to the oscillator.

Since the neutron flux from both a fixed neutron source and from the reactor itself appears as random noise, i.e. both the frequency of occurrence and the frequency of detection are random, the imposition of a signal of fixed frequency should cause a large signal amplitude to occur at that frequency. Due to the characteristics of the reactor the amplitude of the signal varies with both the frequency of the signal and with the reactivity of the reactor. By fixing the frequency of the input signal, the amplitude varies only with the reactivity of the reactor.

Although methods exist for finding the subcritical reactor transfer function, these methods depend on the use of elaborate equipment such as a neutron generator or a

digital computer for the analysis of noise data. The purpose here was to investigate whether a relatively simple and inexpensive method would yield good results.

## THEORY

## Reactor Subcritical Transfer Function

A zero power nuclear reactor may be thought of as being a servo component in a servomechanism which also includes a control rod movement as the input and a neutron flux as the output. The concept of servomechanisms applied to power reactors include such components as heat production, turbine load, poisoning effects and heat transfer characteristics.

For the simplest cases, i.e. for the zero power reactor or for the subcritical reactor, the reactor may be pictured as the open loop servomechanism shown in Fig. 1, where the control rod serves to give the desired input in reactivity,  $\delta k$ , and the reactor responds by yielding some change in neutron output,  $\delta n$ .

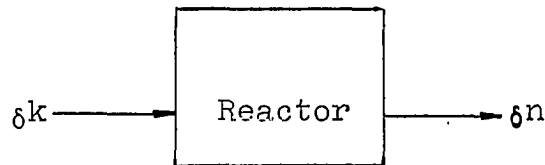


Fig. 1. Reactor as open-loop servomechanism

The ratio of the Laplace transform of the output to the Laplace transform of the input,  $\frac{\delta n(s)}{\delta k(s)}$ , is defined as the "transfer function" and is usually called  $G(s)$  (17). The transfer function  $G(s)$  is a property of the element (reactor) only, and for linear systems is independent of the driving

function and the initial conditions.

The transfer function is normally used to determine whether a system is stable or not.

In order to develop an expression for the transfer function of a system it is necessary to start with the differential equation which relates its input-output characteristics.

The reactor kinetic equations for a subcritical system are (22)

$$\frac{dn}{dt} = [k_{eff}(1-\beta)-1] \frac{n}{\lambda} + \rho e^{-\beta_g^2 \tau} \sum_{i=1}^6 \lambda_i C_i + S \quad (1)$$

$$\frac{dC_i}{dt} = \frac{\beta_i k_{eff} n}{\lambda \rho e^{-\beta_g^2 \tau}} - \lambda_i C_i \quad i = 1, 2, \dots, 6 \quad (2)$$

obtained from diffusion theory for a finite reactor, assuming that the prompt and delayed neutrons have the same energy.

Two transfer functions may be considered, 1) the variation in neutron population due to fluctuations in neutron source strength, and 2) the variation due to changes in  $k_{eff}$ . Because the input of the source strength in this case will appear at random frequencies, only the transfer function which depends on variation in  $k_{eff}$  is important.

$$\text{Let } C_i = C_{i0} + \delta C_i$$

$$S = S_0$$

$$k_{eff} = k_0 + \delta k$$

$$n = n_0 + \delta n$$



then Equation 1 becomes

$$\frac{dn}{dt} = [(k_0 + \delta k)(1 - \beta) - 1] \frac{n_0 + \delta n}{l} + pe^{-B_g^2 \tau} \sum_{i=1}^6 \lambda_i (C_{i0} + \delta C_i) + S_0 \quad (3)$$

and (2) becomes

$$\frac{dC_i}{dt} = \frac{dC_{i0}}{dt} + \frac{d\delta C_i}{dt} = \frac{\beta_i (k_0 + \delta k)}{l pe^{-B_g^2 \tau}} (n_0 + \delta n) - \lambda_i (C_{i0} + \delta C_i) \quad (4)$$

in the steady state condition

$$\frac{dC_{i0}}{dt} = 0 = \frac{\beta_i k_0}{l pe^{-B_g^2 \tau}} n_0 - \lambda_i C_{i0}$$

then

$$\frac{d\delta C_i}{dt} = \frac{\beta_i}{l pe^{-B_g^2 \tau}} [\delta k n_0 + k_0 \delta n] - \lambda_i \delta C_i \quad (5)$$

where  $\delta k$   $\delta n$  is ignored because it is a second order differential. Now substituting  $\lambda_i (C_{i0} + \delta C_i)$  from (4) into (3) and expanding

$$\begin{aligned} \frac{dn_0}{dt} + \frac{d\delta n}{dt} &= \frac{[(k_0 + \delta k)(1 - \beta) - 1]}{l} (n_0 + \delta n) + pe^{-B_g^2 \tau} \sum_{i=1}^6 \\ &\quad \left[ \frac{\beta_i (k_0 + \delta k)(n_0 + \delta n)}{l pe^{-B_g^2 \tau}} - \left( \frac{dC_{i0}}{dt} + \frac{d\delta C_i}{dt} \right) \right] + S_0 \end{aligned} \quad (6)$$

but

$$\frac{dn_0}{dt} = 0 = \frac{[k_0(1 - \beta) - 1]n_0}{l} + pe^{-B_g^2 \tau} \sum_{i=1}^6 \left[ \frac{\beta_i k_0 n_0}{l pe^{-B_g^2 \tau}} - \frac{dC_{i0}}{dt} \right] + S_0$$

then

$$\frac{d\delta n}{dt} = \frac{[(k_0 + \delta k)(1-\beta) - 1]}{\ell} \delta n + \frac{n_0 \delta k (1-\beta)}{\ell} + pe^{-B_g^2 \tau} \sum_{i=1}^6 \left[ \frac{\beta_i (\delta k n_0 + k_0 \delta n)}{\ell pe^{-B_g^2 \tau}} - \frac{d\delta C_i}{dt} \right].$$

Again, ignoring second order differentials and rearranging,

$$\begin{aligned} \frac{d\delta n}{dt} = & \frac{1-\beta}{\ell} (\delta n k_0 + n_0 \delta k) - \frac{\delta n}{\ell} + \sum_{i=1}^6 \frac{\beta_i}{\ell} (\delta k n_0 + k_0 \delta n) \\ & - pe^{-B_g^2 \tau} \sum_{i=1}^6 \frac{d\delta C_i}{dt}. \end{aligned} \quad (7)$$

Now, Equations 7 and 5 are Laplace transformed with zero initial conditions, resulting in

$$\begin{aligned} s\hat{\delta n}(s) = & \frac{1-\beta}{\ell} (k_0 \hat{\delta n}(s) + n_0 \hat{\delta k}(s)) - \frac{\hat{\delta n}(s)}{\ell} + \sum_{i=1}^6 \frac{\beta_i}{\ell} (n_0 \hat{\delta k}(s) \\ & + k_0 \hat{\delta n}(s)) - pe^{-B_g^2 \tau} \sum_{i=1}^6 s\hat{\delta C}_i(s) \end{aligned} \quad (8)$$

and

$$s\hat{\delta C}_i(s) = \frac{\beta_i}{\ell pe^{-B_g^2 \tau}} [n_0 \hat{\delta k}(s) + k_0 \hat{\delta n}(s)] - \lambda_i \hat{\delta C}_i(s). \quad (9)$$

The two equations may be combined by solving Equation 9 for  $\hat{\delta C}_i(s)$  and inserting the result into Equation 8, thus

$$\begin{aligned} s\hat{\delta n}(s) = & \frac{1-\beta}{\ell} (k_0 \hat{\delta n}(s) + n_0 \hat{\delta k}(s)) - \frac{\hat{\delta n}(s)}{\ell} + \sum_{i=1}^6 \frac{\beta_i}{\ell} (n_0 \hat{\delta k}(s) + k_0 \hat{\delta n}(s)) \\ & - pe^{-B_g^2 \tau} \sum_{i=1}^6 s \left[ \frac{\beta_i}{\ell pe^{-B_g^2 \tau}} \frac{[n_0 \hat{\delta k}(s) + k_0 \hat{\delta n}(s)]}{s + \lambda_i} \right]. \end{aligned}$$

This expression may be further reduced, and the terms involving  $\hat{\delta n}(s)$  and  $\hat{\delta k}(s)$  may be collected, with the result

$$\hat{n}(s) \left[ s - \frac{1-\beta}{\ell} k_0 + \frac{1}{\ell} - \sum_{i=1}^6 \frac{\beta_i}{\ell} k_0 \left( 1 - \frac{s}{s+\lambda_i} \right) \right] = \hat{k}(s) \left[ \frac{1-\beta}{\ell} n_0 + \sum_{i=1}^6 \frac{\beta_i}{\ell} n_0 \left( 1 - \frac{s}{s+\lambda_i} \right) \right] .$$

From this the reactor transfer function for a subcritical reactor,  $G(s) = \frac{\hat{n}(s)}{\hat{k}(s)}$ , is

$$G(s) = \frac{\hat{n}(s)}{\hat{k}(s)} = \frac{\frac{1-\beta}{\ell} n_0 + \sum_{i=1}^6 \frac{\beta_i}{\ell} n_0 \left( 1 - \frac{s}{s+\lambda_i} \right)}{s - \frac{1-\beta}{\ell} k_0 + \frac{1}{\ell} - \sum_{i=1}^6 \frac{\beta_i}{\ell} k_0 \left( 1 - \frac{s}{s+\lambda_i} \right)} \quad (10)$$

which may be reduced to

$$G(s) = \frac{\frac{n_0}{\ell} \left( 1 - s \sum_{i=1}^6 \frac{\beta_i}{s+\lambda_i} \right)}{s + \frac{1}{\ell} - \frac{k_0}{\ell} \left( 1 - s \sum_{i=1}^6 \frac{\beta_i}{s+\lambda_i} \right)} . \quad (11)$$

For a subcritical reactor in a steady state  $n_0/\ell$  may be replaced by  $S_0/1-k_0$  (18), and the complex variable  $s$  may be replaced by the complex vector  $j\omega$  (17), so that the reactor transfer function may be written as a function of the frequency, where  $\omega$  is the frequency of the variation measured in radians per second. The frequency transfer function for the subcritical reactor becomes

$$G(j\omega) = \frac{\frac{S_0}{1-k_0} \left( 1 - j\omega \sum_{i=1}^6 \frac{\beta_i}{j\omega+\lambda_i} \right)}{j\omega + \frac{1}{\ell} - \frac{k_0}{\ell} \left( 1 - j\omega \sum_{i=1}^6 \frac{\beta_i}{j\omega+\lambda_i} \right)} . \quad (12)$$

It should be noted that the magnitude of the transfer function normally depends on the magnitude of the steady state neutron population, i.e.,  $n_0/\lambda$ . This means that the gain of the transfer function will vary widely with the reactor power level, and this is a difficulty which must be overcome in the design of a reactor control loop. Fortunately this large variation in gain does not exist for the subcritical function, which is seen to depend upon the source strength and the steady-state multiplication factor. Since the source strength is fixed, the amplitude only depends on the multiplication factor.

The transfer function  $G(j\omega)$  is a complex number which consists of a real part indicating the magnitude and an imaginary part indicating the phase angle. Both of these are functions of the frequency  $\omega$  and of the multiplication factor  $k_0$ . The simplest method for representing these is to plot amplitude and phase angle as a function of frequency and  $k_0$  as a parameter; this is the so-called "Bode diagram".

These plots may be generated either by digital computer calculation or by using the asymptotic approximation applied to a lumped parameter transfer function.

For the latter purpose replace  $\sum_{i=1}^6 \beta_i$  by  $\beta$ , the total fraction of delayed neutrons, and  $\sum_{i=1}^6 \lambda_i$  by  $\bar{\lambda}$ , the average

decay rate of the precursors, so that  $\sum_{i=1}^6 \frac{\beta_i}{j\omega + \lambda_i} \approx \frac{\beta}{j\omega + \bar{\lambda}}$  then

$$G(j\omega) = \frac{\hat{\delta n}(j\omega)}{\hat{\delta k}(j\omega)} = \frac{\frac{s_0}{1-k_0} (1 - j\omega (\frac{\beta}{j\omega + \bar{\lambda}}))}{j\omega + \frac{1}{\ell} - \frac{k_0}{\ell} (1 - j\omega (\frac{\beta}{j\omega + \bar{\lambda}}))} \quad (13)$$

### Cross Correlation Method

The Laplace transform is a mathematical tool used to reduce an equation to one of simpler form. The transformed equation is a function of the complex variable  $s$  and it is difficult to relate it directly to measurable quantities; however, it is possible to use a method known as the frequency-analysis method, and widely used in electrical circuit analysis, to obtain identical results. This method is based on the fact that if a sinusoidal driving signal is imposed on a linear circuit then all other signals appearing in the circuit will be sinusoidal and of the same frequency, but usually of amplitude and phase angle different from the driving signal (17).

In the differential equation of the linear system the differential operator  $d^n/dt^n$  is replaced by  $(j\omega)^n$ , where  $j$  is the complex vector and  $\omega$  is the frequency. The result of this replacement is an algebraic equation which will yield the steady state solution of the dependent variable in complex number representation. When a Laplace transformation is applied to the same equation with zero initial conditions, an

algebraic equation results which is of the same order in  $s$  as is the above equation in  $j\omega$ .

If a sinusoidal signal at a certain frequency is put into the reactor the output divided by the input will therefore be the reactor transfer function as a function of that frequency. The input signal is in the form of a sinusoidal variation of  $k$ , the multiplication factor, of known magnitude and frequency, and the output is the measured variation of the neutron flux.

In this experiment the measurement of the neutron flux involved a neutron detector, its associated amplifier, a count rate circuit and another stage of amplification. All of these components transmit not only the wanted signal but also an extraneous noise signal to which each component adds its own noise. This uncorrelated noise can consist of the bombardment noise which is the randomness with which the neutrons are counted. The detected neutrons come directly from fissions but also from events which have taken place at such a previous time that information about the fission process has been removed; the latter also includes neutrons which are due to such non-fission reactions as  $(\gamma, n)$  reactions. Finally, the amplifiers add thermal noise and noise due to external transients (15).

Since in each input only one frequency is being employed, all other frequencies must be excluded. This may be done by

inserting a bandpass filter; however, this will not exclude the noise which appears at the passed frequency and, besides, the filter itself has noise associated with it. In order to exclude the noise, the noisy signal was cross-correlated with another signal of the same frequency and shape and which was noise-free. The development on which this was based follows.

Let the response of the reactor to a sinusoidal variation in reactivity be  $f(t) = N(t) + n_0 + \delta n \sin(\omega t + \theta)$ , where  $N(t)$  is the signal noise, and  $\theta$  is the phase difference between input and output.  $\delta n$  is the varying response, and  $n_0$  is a steady-state component.

If another sine wave is generated by a function generator which is synchronized with the variation in reactivity, then the two sine waves may be cross-correlated.

Let this generated function be  $g(t) = \sin \omega t$ . The cross-correlation for the two functions is (9)

$$\psi_{fg}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t)g(t+\tau)dt. \text{ Here the function}$$

becomes

$$\psi_{fg}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T [N(t) + n_0 + \delta n \sin(\omega t + \theta)] \sin \omega(t+\tau) dt$$

and for  $\tau = 0$  we get

$$\psi_{fg}(\tau=0) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T [N(t) + n_0 + \delta n \sin(\omega t + \theta)] \sin \omega t dt$$

Experimental data are taken for a finite interval of time and at a fixed frequency; if an integral number of cycles is

observed,  $T = A/f$ , where  $A$  is an integer, and  $f$  is the frequency in cycles per second. The mean value of  $N(t)$  is 0, so that the term  $N(t) \sin \omega(t+\tau)$  represents a harmonically modulated noise signal. By expanding the cross-correlation function, inserting the expression for  $T$ , and taking a finite time interval

$$\begin{aligned} \psi_{fg}(0) = & \frac{f}{2A} \int_{-A/f}^{A/f} N(t) \sin \omega t \, dt + \frac{f}{2A} \int_{-A/f}^{A/f} n_0 \sin \omega t \, dt \\ & + \frac{f}{2A} \int_{-A/f}^{A/f} \delta n \sin(\omega t + \theta) \sin \omega t \, dt \end{aligned}$$

Since  $n_0$  is constant the first two terms are equal to zero and

$$\begin{aligned} \psi_{fg}(0) &= \frac{f}{2A} \int_{-A/f}^{A/f} \delta n \sin(\omega t + \theta) \sin \omega t \, dt \\ &= \frac{f}{A} \int_0^{A/f} \delta n \sin(\omega t + \theta) \sin \omega t \, dt \\ &= \frac{f}{A} \int_0^{A/f} \frac{\delta n}{2} [\cos(\omega t + \theta - \omega t) - \cos(\omega t + \theta + \omega t)] dt \end{aligned}$$

which reduces to

$$\psi_{fg}(0) = \frac{f}{A} \frac{\delta n}{2} \left[ \frac{A}{f} \cos \theta - \frac{\cos \theta}{2\omega} \sin 2\omega \frac{A}{f} - \frac{\sin \theta}{2\omega} (\cos 2\omega \frac{A}{f} - 1) \right]$$

Since  $\omega = 2\pi f$ , and  $A$  is an integer the function becomes

$$\psi_{fg}(0) = \frac{\delta n}{2} \cos \theta . \quad (14)$$

Now, if we let  $\tau_1 = 1/(4f)$ , and solve for the cross-correlation function we get



$$\psi_{fg}(\tau_1) = \frac{f}{A} \int_0^{A/f} \delta n \sin(\omega t + \theta) \sin \omega(t + \tau_1) dt$$

but  $\sin \omega(t + 1/(4f)) = \cos \omega t$ , so that

$$\begin{aligned} \psi_{fg}(\tau_1) &= \frac{f}{A} \int_0^{A/f} \delta n \sin(\omega t + \theta) \cos \omega t dt \\ &= \frac{f}{A} \int_0^{A/f} \frac{\delta n}{2} [\sin(\omega t + \theta + \omega t) + \sin(\omega t + \theta - \omega t)] dt \end{aligned}$$

which reduces to

$$\begin{aligned} \psi_{fg}(\tau_1) &= \frac{f}{A} \left[ \frac{\delta n}{4\omega} (-\cos \theta (\cos 2\omega \frac{A}{f} - 1) + \sin \theta \sin 2\omega \frac{A}{f}) \right. \\ &\quad \left. + \frac{\delta n}{2} \sin \theta \right] \\ \psi_{fg}(\tau_1) &= \frac{\delta n}{2} \sin \theta . \end{aligned} \tag{15}$$

The wiring diagram showing how the two cross-correlation functions were obtained experimentally is shown in Fig. 2. The two functions generated by the integrators are not identical to (14) and (15) because the factor  $f/A$  does not appear in front of the integral. The experimentally determined cross-correlation functions are therefore equal to  $A/f \delta n/2 \cos \theta$  and  $A/f \delta n/2 \sin \theta$  respectively. Again, since  $A/f$  is equal to the time over which the data are taken, the two integrator outputs are

$$\begin{aligned} X &= T \frac{\delta n}{2} \cos \theta \text{ and} \\ Y &= T \frac{\delta n}{2} \sin \theta . \end{aligned}$$

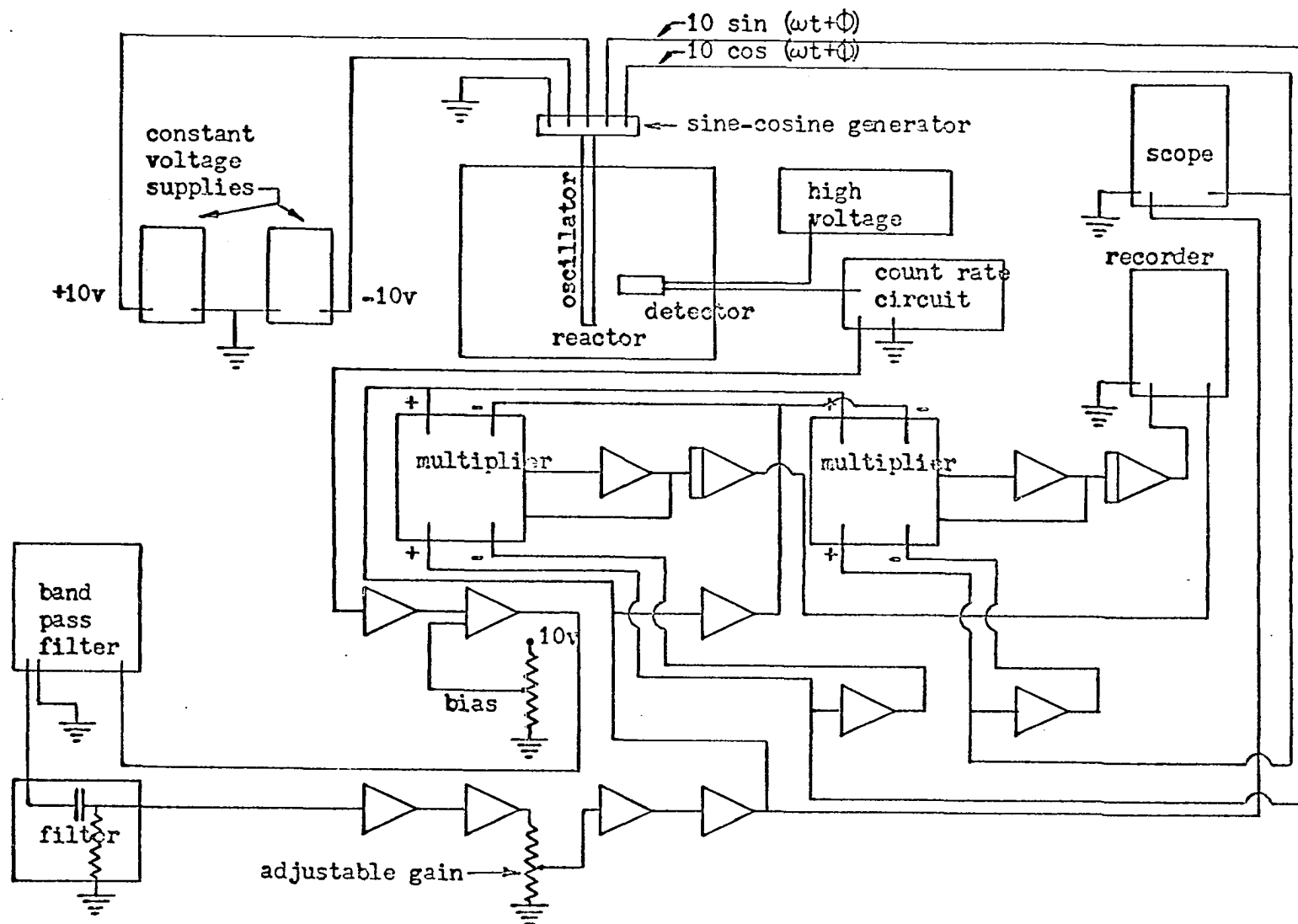


Fig. 2. Schematic wiring diagram of experiment

It is apparent that the output of the integrators increase in direct proportion to the duration of the experiment.

Several possibilities exist for obtaining the phase angle and the amplitude of the transfer function from the measured signals X and Y. The method used here was to record the two signals and to carry out the calculations subsequently. Another method would be to feed the signals into an analog computer and get the answer directly.

## PERFORMANCE OF EXPERIMENT

The experiment required varying the reactivity of the reactor by means of an oscillator. The oscillator, however, also represented a mean net amount of negative reactivity which might influence the normal operation of the reactor. The desired end result was to find the transfer function at a fixed frequency as a function of reactivity, but the reactivity had to be measured by some other method.

The reactor control rods may be calibrated in such a manner that any given position of the rods results in a known amount of inserted reactivity, but the presence of the additional absorber in the oscillator might also change the worth of the control rods. Before the experiment it was therefore necessary to calibrate the control rods. This was done by the standard positive period method (11) and the result for the regulating rod is shown in Fig. 3. It should be noted that there was no discernible difference between the worth of the regulating rod with and without the oscillator in the reactor. A similar curve is shown for the shim rod in Fig. 4.

In order to measure the phase angle between the input and the output, it was necessary to know the angular position of the oscillator. This position would then have to correspond to the angular position of the sine wave generated by the potentiometer. The potentiometer was

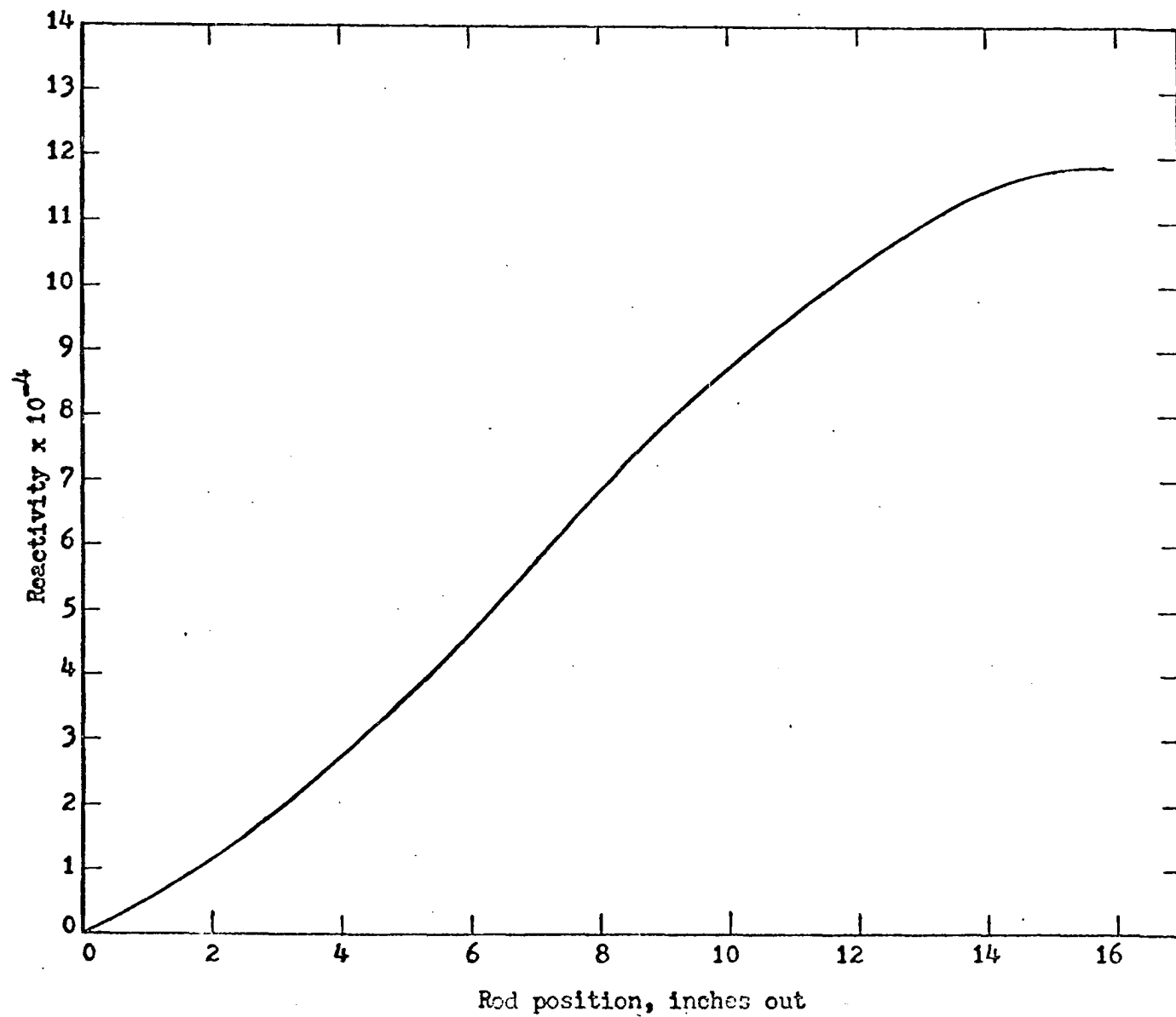


Fig. 3. Regulating rod calibration curve

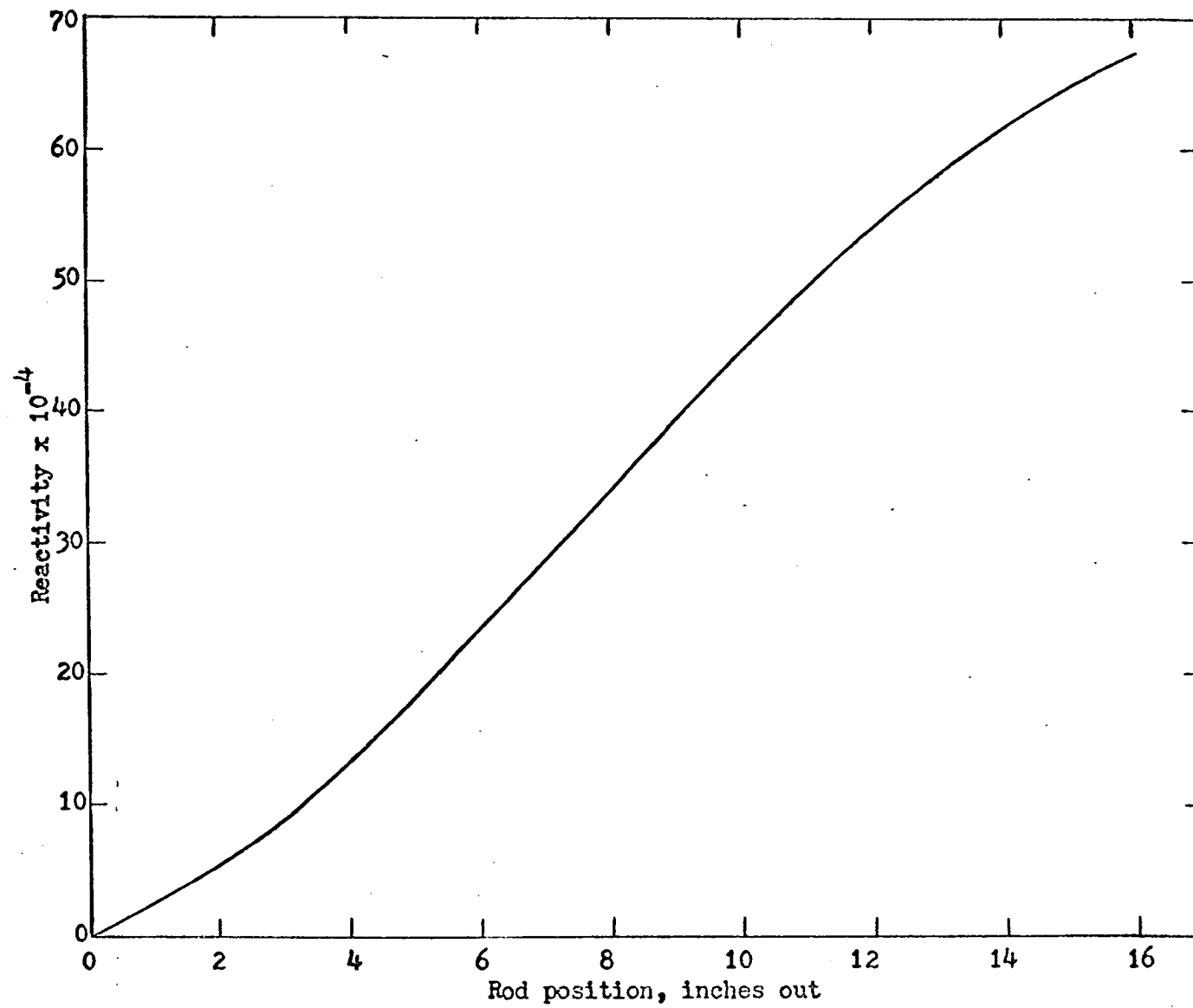


Fig. 4. Shim rod calibration curve

attached on the end of the oscillator, with the two shafts coupled together, but in such a manner that the body of the potentiometer could be rotated  $360^\circ$ . A schematic drawing of the experimental apparatus is shown in Fig. 5. With the reactor critical at a low power level (0.1 watt) under automatic control, it was possible to determine that position of the oscillator which resulted in maximum absorption by observing the minimum control rod position. A voltage source was connected to the potentiometer, and with the oscillator stationary, the relative position of the stator and rotor of the sine pot was adjusted to give a maximum voltage output. When the maximum voltage was reached, the potentiometer body was locked in place.

The reactivity worth of the oscillator was then calibrated for approximately every 10 degrees of rotation by means of the regulating rod position. The rod position was subsequently converted to reactivity by using Fig. 3. The result is shown plotted in Fig. 6. The amplitude of the sine curve generated by the potentiometer is arbitrary since it depends on the applied voltage.

The detector which was placed in the reactor to measure the effects of the oscillating reactivity was an ordinary  $\text{BF}_3$  detector which indicates single events as voltage pulses. When the neutron flux varies the rate of appearance of the voltage pulses varies also. By putting the pulses from the

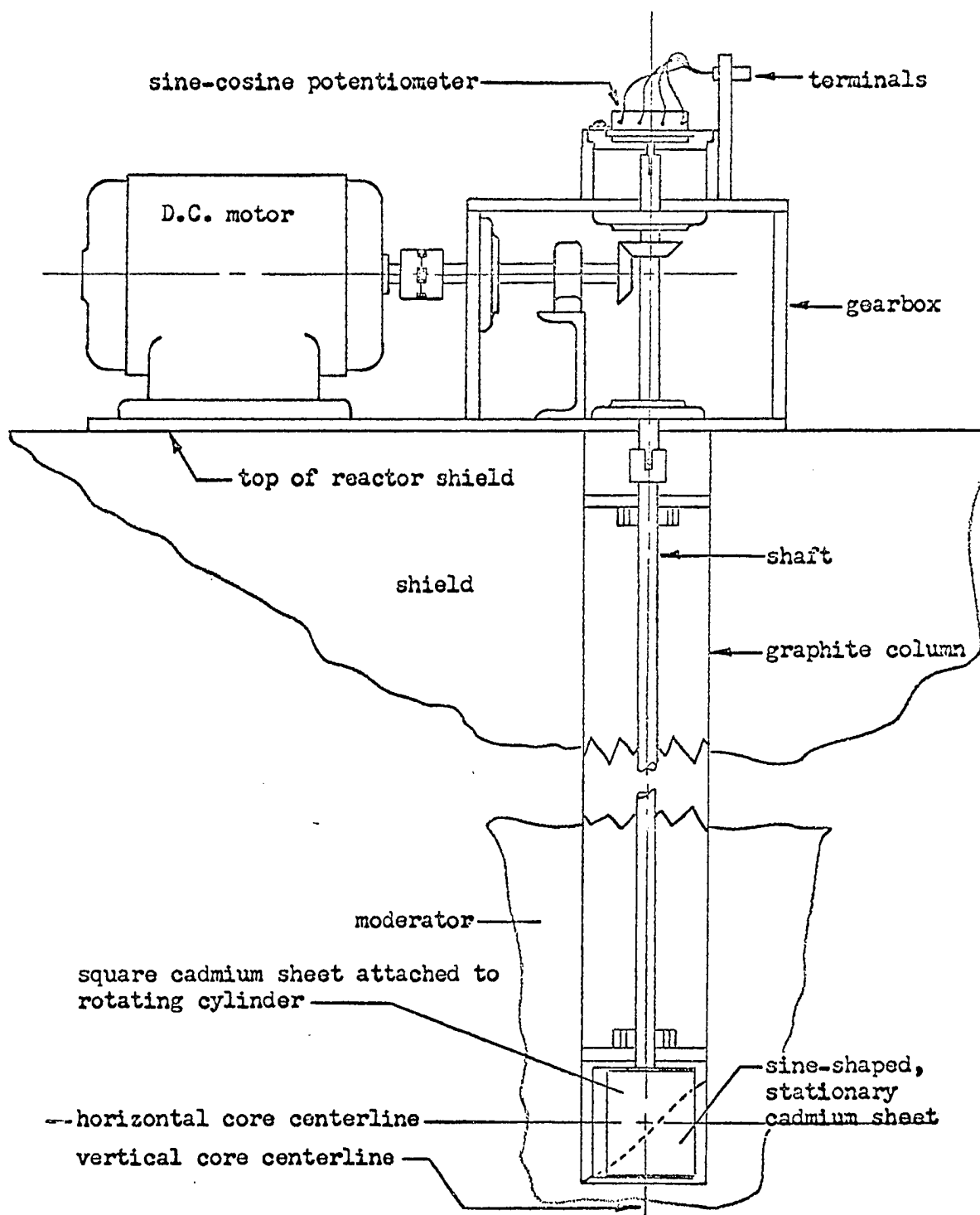
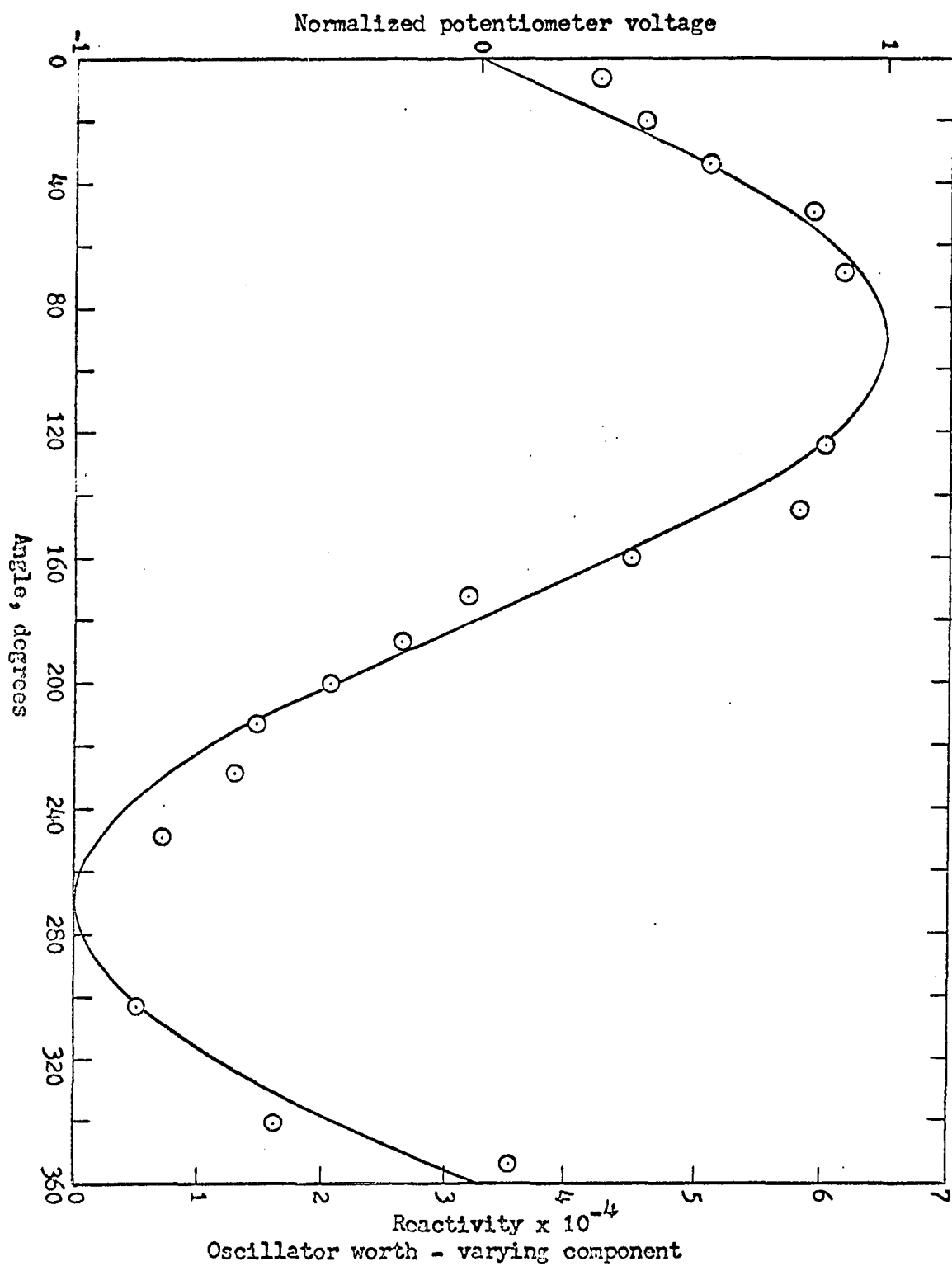


Fig. 5. Cross-section of oscillator and drive



Fig. 6. Varying component of oscillator worth and potentiometer voltage vs.  
angular position



detector through a count rate circuit, a voltage results which varies directly with the rate of appearance of the pulses. Thus, for a pure sinusoidal variation in flux, the count rate circuit would yield a sinusoidal variation in voltage. Because the flux actually varies about an average value so will the voltage vary about an average value. An oscilloscope was employed to watch the waveform at each step throughout the entire experiment, and the scope made it possible to apply a bucking voltage to the output from the count rate circuit so that the net output was a waveform with an average value of zero. Although this step was not necessary, it was desirable to have as pure a signal as possible enter the integrators.

The output from the summing junction for the count rate circuit and the bucking voltage, although now averaging about a zero value, still contained information about all frequencies and was not sinusoidal. The signal was next put into a band-pass filter which had both its high pass and its low pass frequencies set at the same frequency as the frequency with which  $\delta k$  was varied (10 cps). The output from the band-pass filter when displayed on the scope was obviously a sine wave although noise was still present. The bandpass filter also added a slight D.C. component which was removed by going through a high pass filter consisting of a resistor and a condenser.

The signal was finally amplified through several stages of amplification until it was approximately of the same size as the signal from the potentiometer. With both signals displayed simultaneously they were adjusted until they were near the 10 volt limit determined by the analog computer equipment. The input voltage which was thus established for the potentiometer and the amplification established for the  $\text{BF}_3$  detector signal were unchanged for the duration of the experiment.

In order to establish whether the noise would indeed be eliminated from the signal by the cross-correlation procedure, the  $\text{BF}_3$  detector was located near a neutron source which gave off  $\sim 2.5 \times 10^5$  counts/minute. The signal obtained from this source was passed through all of the equipment outlined and shown in Fig. 2. The oscillator was started, but only for the purpose of driving the potentiometer, and the voltage established above was applied to it. The potentiometer had two output voltages which were sine waves  $90^\circ$  apart, and each of these waves was multiplied by the filtered and amplified output from the  $\text{BF}_3$  detector. Since there should be no correlation at all between the signal from the source and the signals from the potentiometer, the cross-correlation function should be zero for both sets of waves.

Fig. 7 shows a pair of typical recordings which were obtained by cross-correlating the two sine waves with a noise signal which contained no information. Detailed

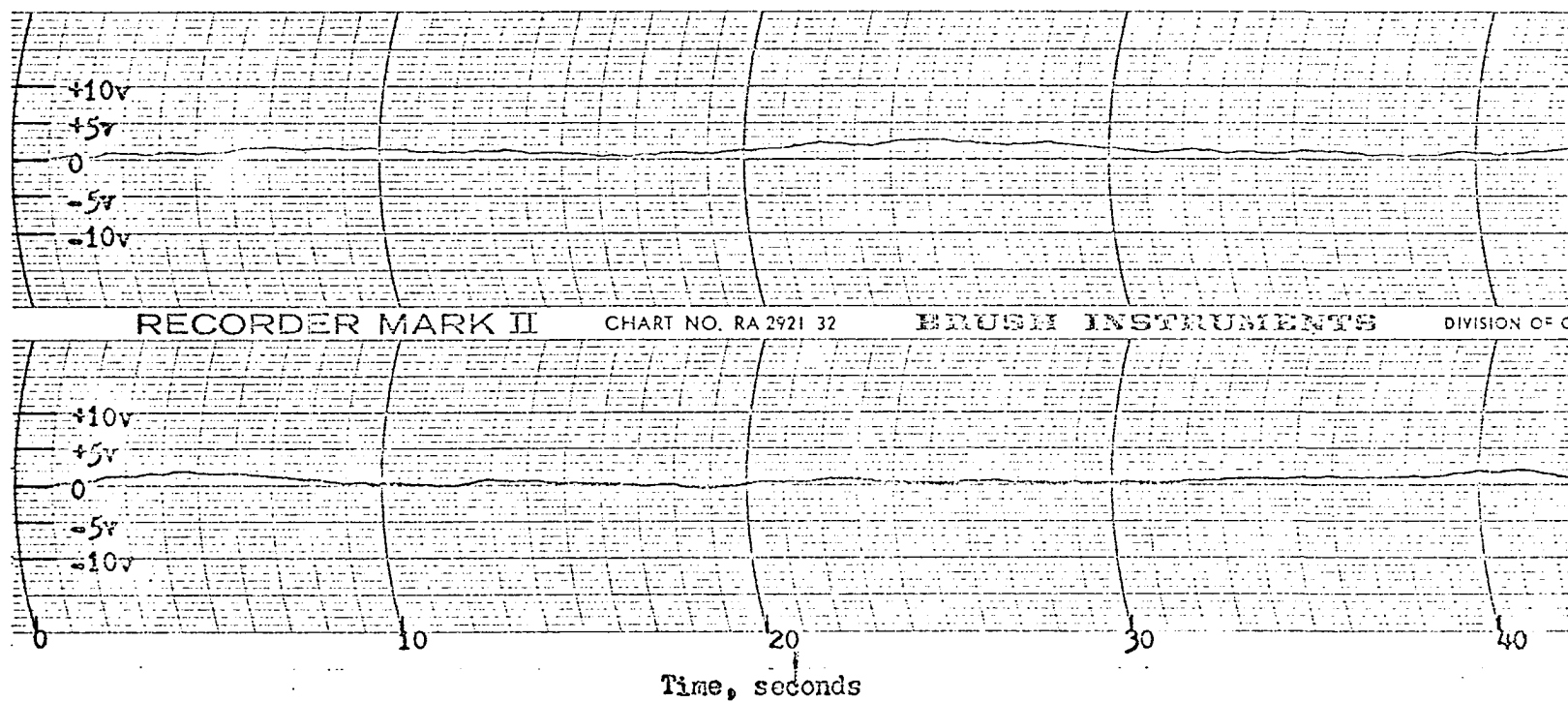


Fig. 7. Cross-correlation of sine wave and uncorrelated signal

examination of the recordings showed that the time average of all of these signals is zero, but also that the statistical variation of the noise gave rise to the possibility of a signal being present at any given time. This result showed that in order to get meaningful data from the experiment it would be necessary to obtain a number of recordings for each given condition of the reactor, and to analyze all of these recordings in order to arrive at a representative value for each condition.

It should finally be mentioned that although the magnitude of the transfer function was one of the desired results, and that it could have been calculated because the variations in  $\delta k$  was known and  $\delta n$  was measured, it was only necessary to know the relative magnitude and the result could be normalized with respect to any desired point.

## METHOD OF MEASUREMENT

The reactor was brought to critical at a power level of  $3.75 \times 10^{-3}$  watts; this was done with the oscillator rotating because only then could it be established with certainty that the average reactivity of the oscillator was inserted. The small sinusoidal disturbance introduced by the oscillator had no effect on the operation of the reactor. The reactor mode of operation was manual so that the control rod positions could be determined from the reactor panel. When the reactor is critical, the two safety rods are always fully withdrawn from the core, while the shim and regulating rod positions can be adjusted to keep the multiplication factor at unity. In this case the regulating rod was fully withdrawn (16 inches out), while the shim rod was withdrawn sufficiently to establish the critical condition.

The regulating rod was inserted six inches, whereupon the neutron source was also inserted to maintain a steady-state condition.

With an external source present the flux in a reactor, as a function of time, has the form (8)

$$\phi(t) = C \frac{p}{k \Sigma_a} \sum_{n=1}^{\infty} \left[ A_n e^{(k_n - 1) \frac{t}{l_n}} + \frac{k e^{-B_n^2 \tau} S_n}{(1 + L^2 B_n^2)(1 - k_n)} \right]$$

where  $C$  is a variable depending on position in the reactor, and  $A_n$  is a series of constants corresponding to the eigenvalues  $B_n^2$ .

The first term inside the brackets is the term due to fissioning and is an exponential in time; it increases or decreases depending on whether  $k_n$  is greater or smaller than one. If all  $k_n$  are smaller than one the flux will obviously decay exponentially to zero at a rate which is determined by the size of  $k_n$ . The second term inside the brackets is the source term and is constant for all values of  $k_n$  less than one. Thus, if operation at a steady state below  $k_n = 1$  is desired, it is necessary to introduce the external source.

When  $k_n = 1$  it is seen that the first term inside the brackets is constant whereas the second term becomes infinite. For critical operation it is therefore necessary to have the external source removed.

The initial step of introducing the source and the six inches of regulating rod therefore established the reactor subcritical at a steady state by the amount given by the reactivity equivalent of the first six inches of regulating rod. This amount of rod was equivalent to a negative reactivity of only  $3.11 \times 10^{-4}$ . The addition of the source had the effect of increasing the flux level, but because the reactor was now subcritical it occurred at a continuously decreasing rate. When the reactor finally established its equilibrium, the power level was  $3.94 \times 10^{-2}$  watts, about 10 times greater than the initial critical level. Because it was feared that the  $\text{BF}_3$  detector would saturate at a higher count rate, the reactor subcritical transfer function was not determined



for smaller values of negative reactivity.

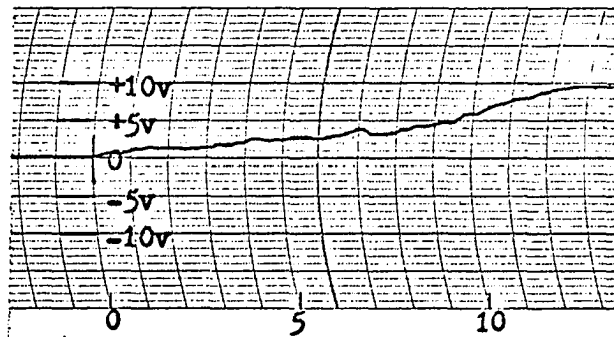
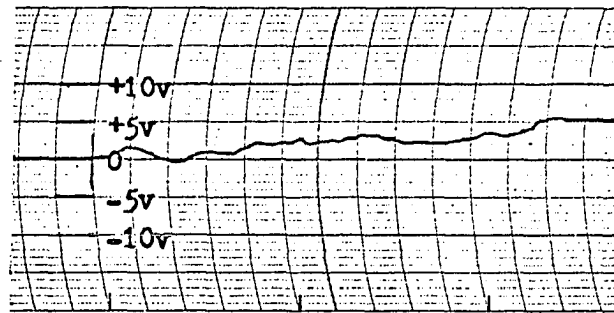
When the flux had reached its equilibrium value, the integration was started with a two-pen recorder continuously charting both of the cross-correlation functions. When one of the integrals reached the 10-volt saturation level of the analog computer, the integrator was turned off, and the final voltage was noted for each channel. The process of integration was repeated at the same flux level until sufficient records were obtained for determination of a statistical spread.

The regulating rod was next inserted an additional two inches, representing a reactivity of approximately  $2 \times 10^{-4}$ , and after the reactor had reestablished its equilibrium, the integration process was carried out at the new flux level. A record was also kept of the power level shown on the reactor panel and of the reactor coolant inlet and outlet temperatures.

With the completion of each run the regulating rod was inserted an additional two inches and the process repeated until the rod was fully inserted. When the regulating rod had been fully inserted, the process was repeated with the shim rod in steps of two inches at a time. Finally, measurements were made on the reactor with the safety rods inserted one at a time by deactivation of their magnetic clutches. The last measurement was thus performed with

the maximum available shutdown margin without removing the moderator, and corresponding to an inserted reactivity of  $2.36 \times 10^{-2}$ .

As a result of the procedure described, a record was established of the two cross-correlation functions as functions of the rod positions with the oscillator rotating at 10 cycles per second. Sample recordings are shown in Figs. 8 and 9, for  $k_0 = 0.99894$  and  $k_0 = 0.97936$  respectively. The top trace in each recording represents  $T \frac{\delta n}{2} \cos \theta$  vs.  $T$  and the bottom trace in each represents  $T \frac{\delta n}{2} \sin \theta$  vs.  $T$ .



Time, seconds

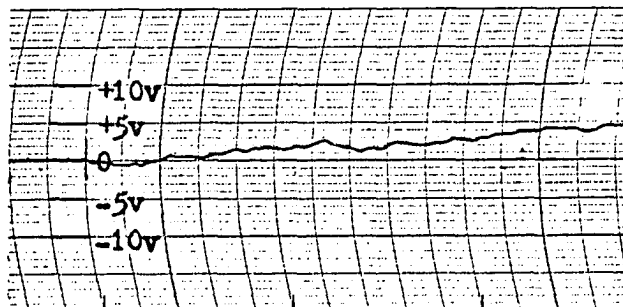
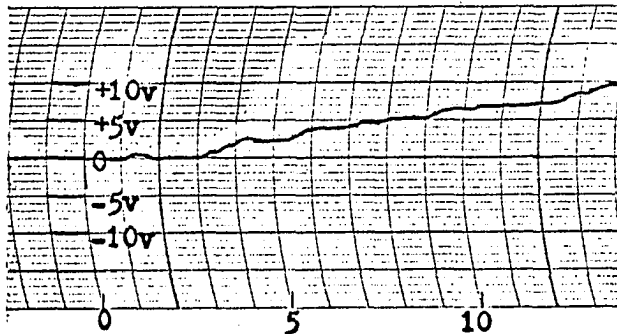


CHART NO. RA 2921 32

BRUSH INST



Time, seconds

Fig. 8. Typical recordings at small negative reactivity

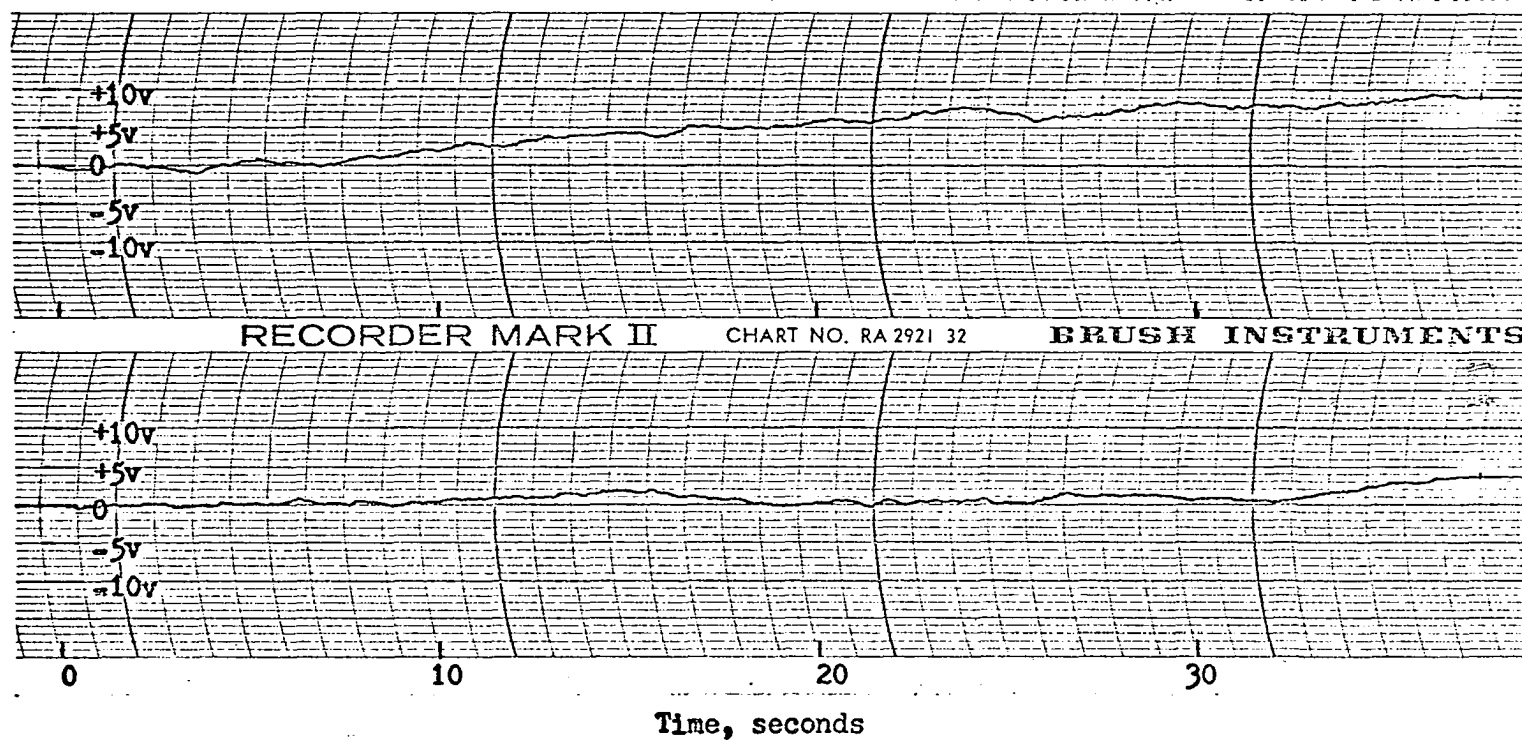


Fig. 9. Typical recording at large negative reactivity

## CALCULATIONS

## Reduction of Experimental Data

The experimental data were converted to the desired form by first changing all of the rod positions to equivalent inserted reactivity. This was done by using Figs. 3 and 4 for the regulating and shim rods respectively. The equivalent reactivity of the safety rods was obtained from earlier determinations of their values.<sup>1</sup>

Next it was desired to get the transfer function from the cross-correlation functions. The cross-correlation functions were seen earlier to be of the form

$$X = T \frac{\delta n}{2} \cos \theta \quad \text{and}$$

$$Y = T \frac{\delta n}{2} \sin \theta .$$

One purpose of the cross-correlation technique was to eliminate all of the errors introduced by noise, but since it was not possible to exceed a value of 10 volts on the analog computer, X or Y, whichever was the larger, was limited to 10 volts. Since both equations are linear with respect to T, where T is time in seconds, it is seen that for some values of  $\delta n$  and  $\theta$  the limiting value could be reached in a few seconds. The ability of the cross-correlation function to eliminate noise errors depends on

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<sup>1</sup>Recorded in UTR-10 reactor manual.

the integration to take place for a long time. The fact that the maximum value of one of the integrators in some instances was reached in a short time therefore decreased the accuracy of that particular integral, and it was necessary to average many such integrals. The process of averaging has the same effect as a long integration time, since a random noise signal is as likely to add to the integral as it is to subtract from it.

From the expression of X and Y it is seen that they are linear functions of the independent variable and that they pass through the origin. The obvious approach to use on linear data with random errors is to divide the axis of the independent variable into segments, read the value of the dependent variable at each division, and subject the resulting data to a least squares fit. The data collected here are, however, of a different nature (1), because although the error from the detector signal is completely random, this signal was fed to an integrator, and it was the result of the integration over time which was obtained as final data; this means that any error in the final signal at any given time will depend in magnitude on all of the errors which preceded it in time and it is evident that the process of dividing the axis of the independent variable into segments will reduce the amount of information contained in the data as the origin is approached. This of course is also a direct consequence of the fact that the integration should

be carried out over infinite time in order to remove the noise errors. With the above in mind the most accurate method of evaluation is to measure the final value of the integral and divide it by the integration time<sup>1</sup>, to yield  $\Delta_{\cos} = \frac{\delta n}{2} \cos \theta$  and  $\Delta_{\sin} = \frac{\delta n}{2} \sin \theta$  from the two sets of data.

The calculations performed on the recorded data were thus to find the slope of each of the recordings, and to average the slopes for each set of equal reactor conditions. The average gave the best value of  $\frac{\delta n}{2} \sin \theta$  and  $\frac{\delta n}{2} \cos \theta$  at that condition, with the variance of the individual recordings given by  $V_i = \frac{\Sigma(\Delta_i - \bar{\Delta})^2}{n - 1}$ , where  $\Delta_i$  is the slope of the individual recording,  $\bar{\Delta}$  is the average slope, and  $n$  is the number of observations. The variance of the mean for the set is given by  $V = \frac{1}{n} \frac{\Sigma(\Delta_i - \bar{\Delta})^2}{n - 1}$ , and the standard deviation is given by

$$\bar{s} = \sqrt{\frac{\Sigma(\Delta_i - \bar{\Delta})^2}{n(n-1)}}.$$

With these calculations all of the slopes,  $\bar{\Delta} \pm \bar{s}$ , were obtained for both the cosine and the sine functions. The phase angle  $\theta$  and its uncertainty was then found from

$$\theta \pm \delta\theta = \tan^{-1} \frac{\bar{\Delta}_{\sin} \pm \bar{s}_{\sin}}{\bar{\Delta}_{\cos} \pm \bar{s}_{\cos}}$$

which may also be written as

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<sup>1</sup>David, Dr. Herbert, Ames, Iowa. Analysis of continuous data. Private communication. 1967.

$$\theta \pm \delta\theta = \tan^{-1} \frac{\bar{\Delta}_{\sin}}{\bar{\Delta}_{\cos}} \pm \frac{\bar{\Delta}_{\sin}\bar{\Delta}_{\cos}}{\bar{\Delta}_{\sin}^2 + \bar{\Delta}_{\cos}^2} \left( \frac{\bar{s}_{\sin}}{\bar{\Delta}_{\sin}} + \frac{\bar{s}_{\cos}}{\bar{\Delta}_{\cos}} \right)$$

where the maximum value of  $\delta\theta$  is obtained by choosing similar signs throughout and is measured in radians.

$\delta n$  and its uncertainty which is a measure of the change in neutron population was found from

$$\delta n \pm \epsilon = \frac{2[\bar{\Delta}_{\sin} \pm \bar{s}_{\sin}]}{\sin(\theta \pm \delta\theta)},$$

where  $\epsilon$  indicates the uncertainty in  $\delta n$ . The uncertainty  $\epsilon$  may also be written as

$$\epsilon = 2 \sqrt{\frac{-2}{\bar{\Delta}_{\sin}^2} + \frac{-2}{\bar{\Delta}_{\cos}^2}} \left[ \frac{\bar{s}_{\sin}}{\bar{\Delta}_{\sin}} + \frac{\bar{\Delta}_{\cos} \delta\theta}{\bar{\Delta}_{\sin}} \right].$$

These calculations were carried out for each reactivity used during the experiment, and the results are shown in Table 1. The phase angle is plotted as a function of the steady state multiplication factor in Fig. 12 and a similar plot for the magnitude of the transfer function is shown in Fig. 13.

#### Calculation of Theoretical Data

The theoretical values of the transfer function were found as functions of frequency for several values of  $k_0$  by using the asymptotic approximation for the frequency analysis (17) and applied to the lumped parameter equation.



Fig. 10. Transfer function amplitude vs. frequency for small negative reactivities

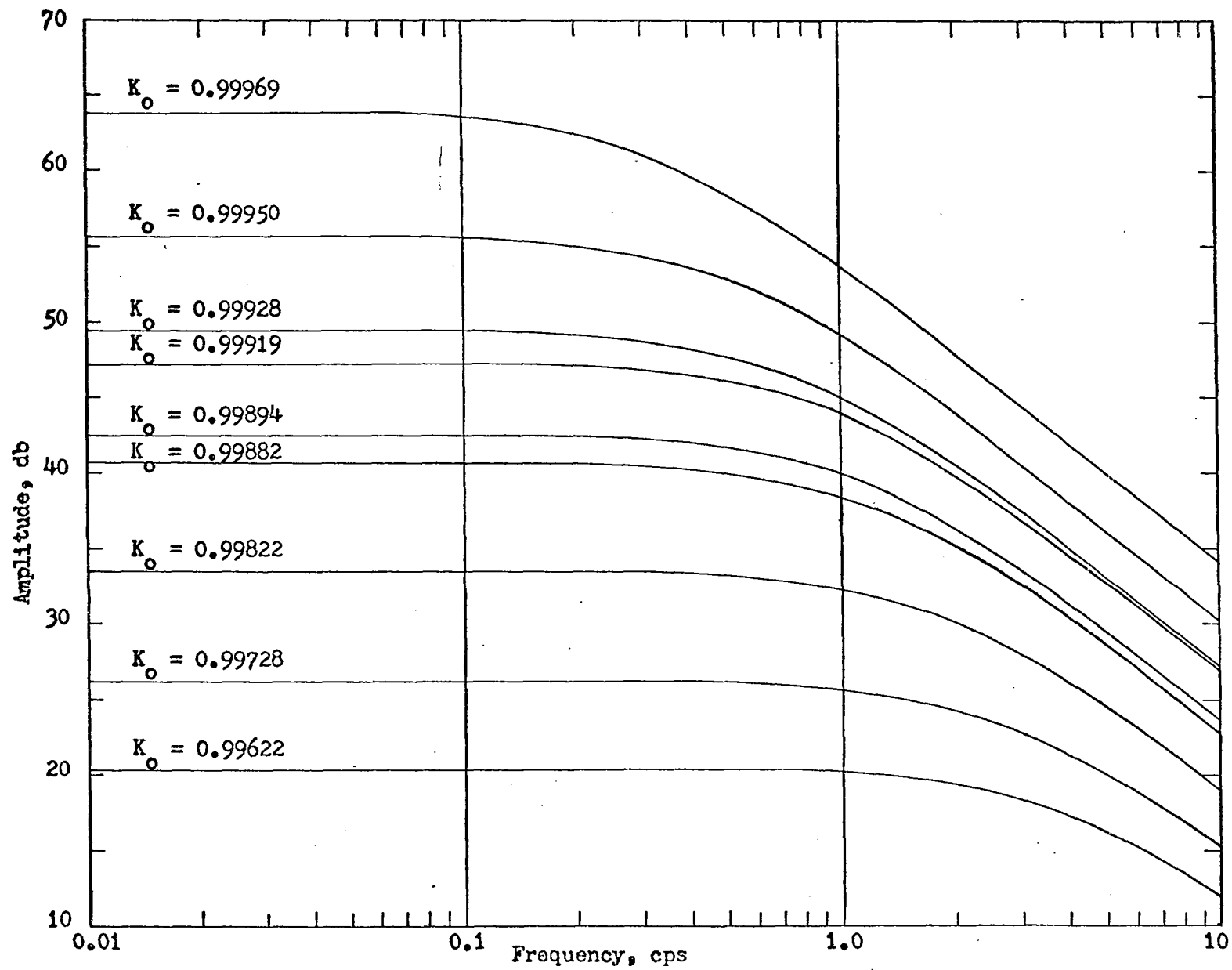
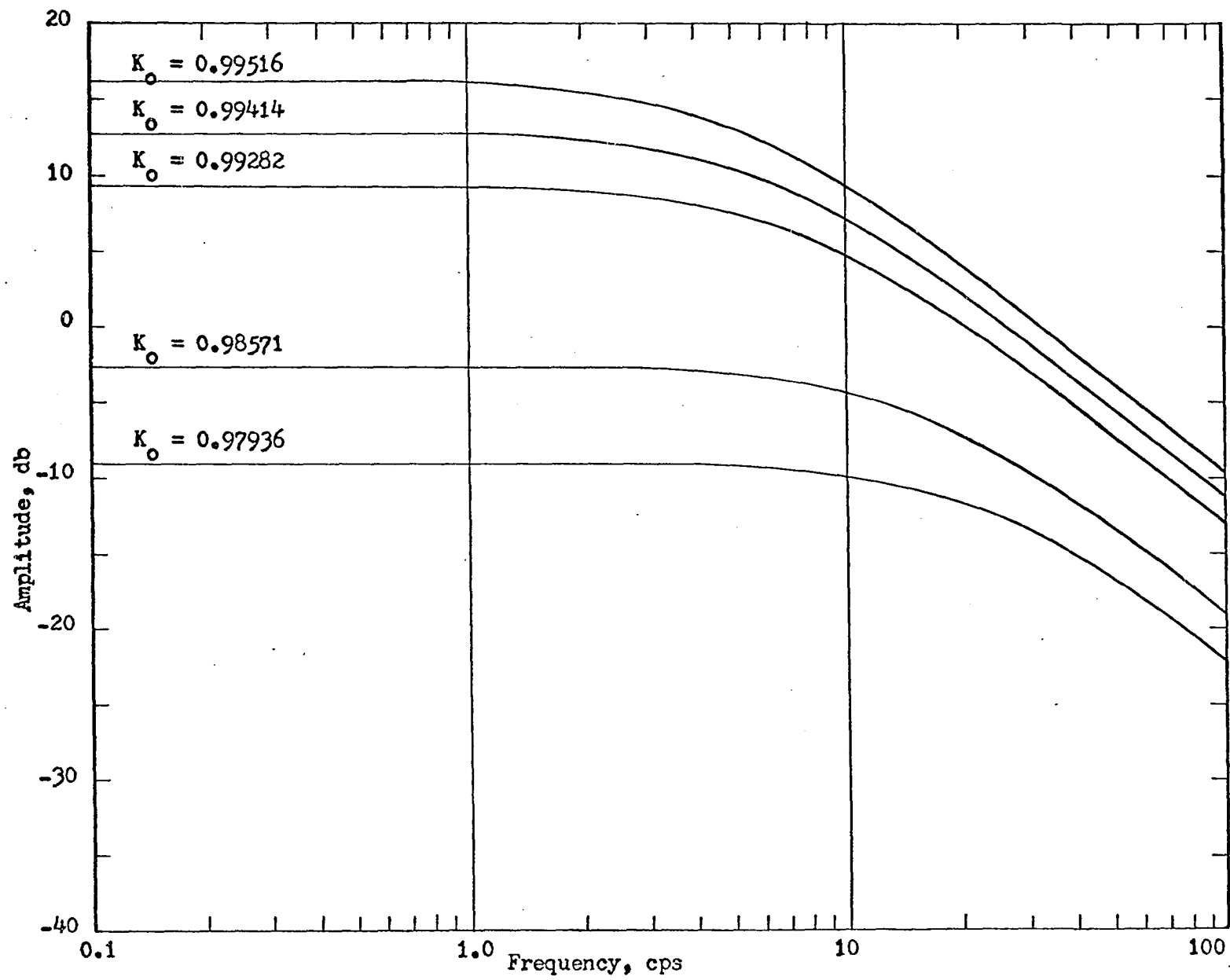


Fig. 11. Transfer function amplitude vs. frequency for large negative re-activities



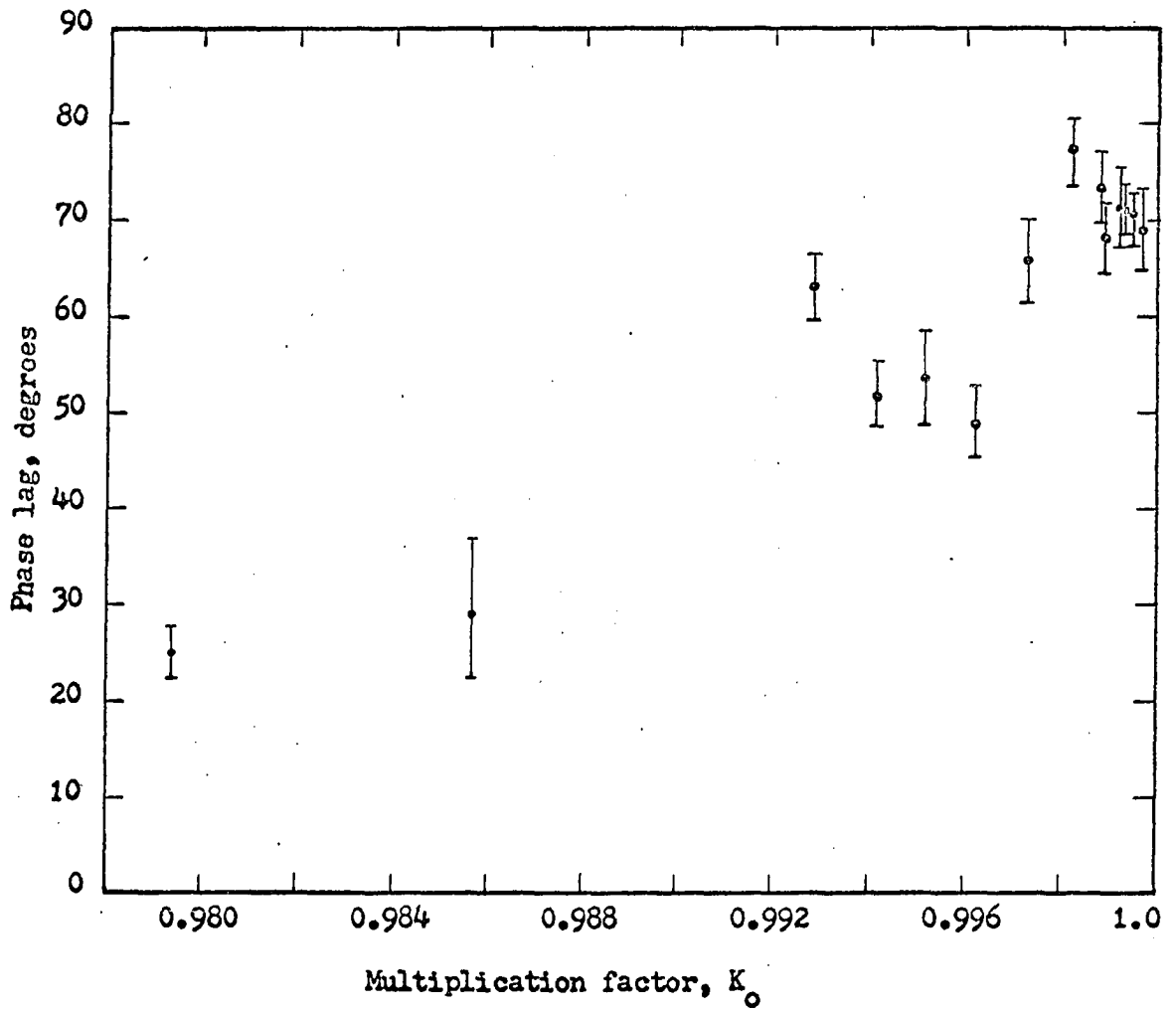


Fig. 12. Experimental values of transfer function phase angle at 10 cps vs. multiplication factor

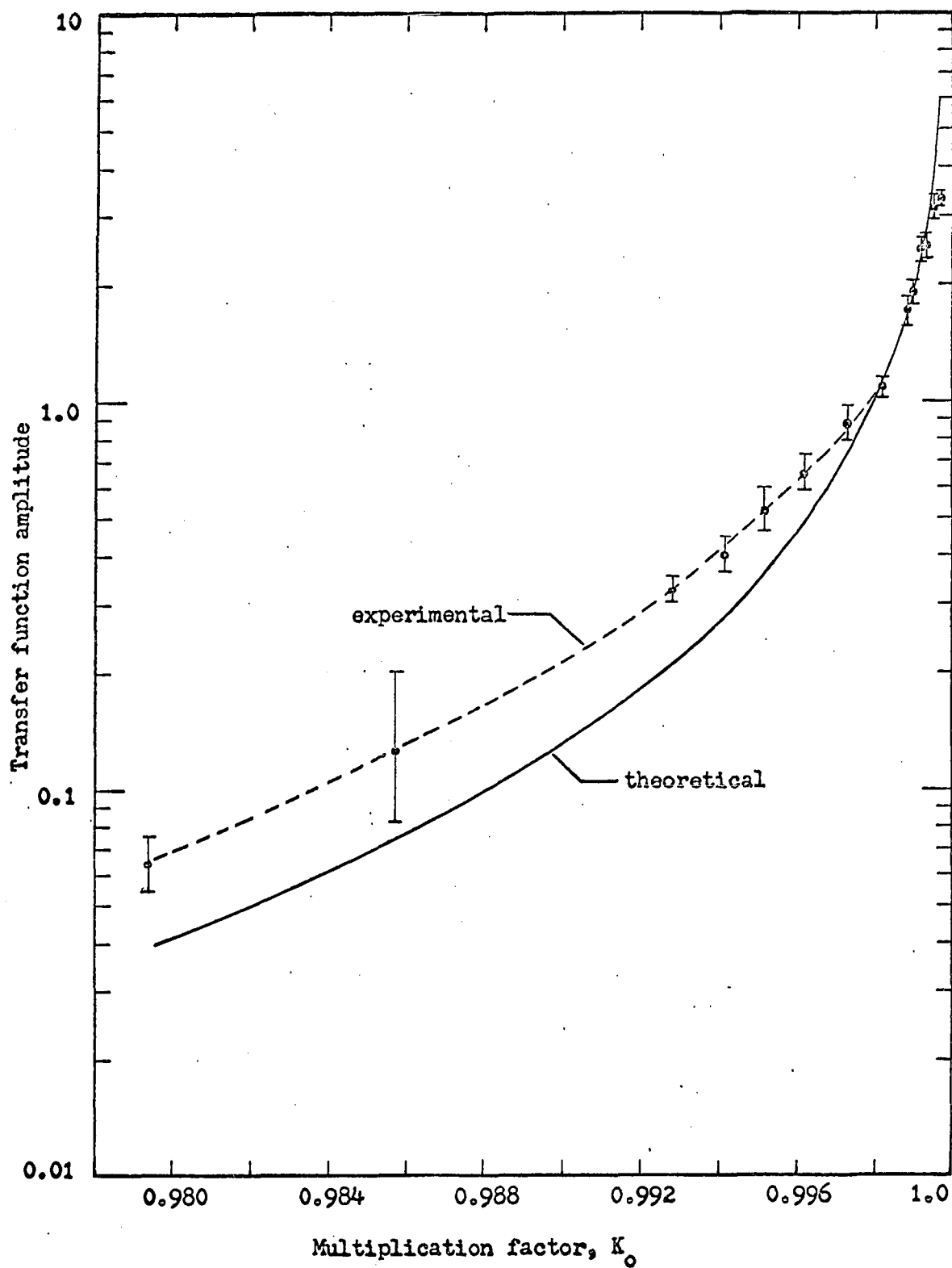


Fig. 13. Experimental and theoretical values of transfer function amplitude at 10 cps vs. multiplication factor.

Table 1. Transfer function magnitudes and phase angles

Run No.	Multipli- cation Factor	Phase angle degrees	Phase angle Uncertainty degrees	Relative Magnitude	Magnitude Uncertainty %
II <sub>a</sub>	0.99969	69.3	± 4.2	3.31	± 4.2
II <sub>b</sub>	0.99950	70.8	± 2.8	3.13	± 7.2
II <sub>c</sub>	0.99928	71.4	± 2.5	2.51	± 7.4
II <sub>d</sub>	0.99919	71.3	± 4.2	2.45	± 6.9
II <sub>e</sub>	0.99894	68.2	± 3.7	1.90	± 6.8
II <sub>f</sub>	0.99882	73.7	± 3.7	1.71	± 8.8
III <sub>a</sub>	0.99822	77.3	± 3.4	1.08	± 6.5
III <sub>b</sub>	0.99728	65.9	± 4.5	0.881	±10.3
III <sub>c</sub>	0.99622	49.0	± 3.8	0.654	±10.6
III <sub>d</sub>	0.99516	53.7	± 5.0	0.524	±13.1
III <sub>e</sub>	0.99414	51.9	± 3.3	0.402	±10.9
III <sub>f</sub>	0.99282	63.1	± 3.3	0.327	± 7.8
IV	0.98571	29.1	± 7.0	0.128	±46.8
V	0.97936	25.0	± 2.7	0.0643	±17.0
II' <sub>a</sub>	0.99916	68.6	± 1.5	2.21	± 7.0
II' <sub>b</sub>	0.99907	62.2	± 4.7	2.01	± 9.7
II' <sub>c</sub>	0.99888	61.8	± 5.5	1.95	±12.6
II' <sub>d</sub>	0.99867	67.7	± 5.3	1.61	±10.0
II' <sub>e</sub>	0.99843	62.6	± 3.4	1.46	± 7.9
III' <sub>a</sub>	0.99764	58.0	± 5.1	0.961	±16.4

The following values apply to the UTR-10 reactor (14a)

$$\bar{\lambda} = 0.08 \text{ sec}^{-1}$$

$$\beta = 0.008$$

$$l = 1.5 \times 10^{-4} \text{ sec}$$

$$\text{Now, if } \omega \gg \bar{\lambda}, \text{ then } G(j\omega) \approx \frac{\frac{S_0}{1-k_0}(1-j\omega(\frac{\beta}{j\omega}))}{j\omega + \frac{1}{l} - \frac{k_0}{l}(1-j\omega(\frac{\beta}{j\omega}))} =$$

$$\frac{\frac{S_0}{1-k_0}(1-\beta)}{j\omega + \frac{1}{l} - \frac{k_0}{l}(1-\beta)} \text{ and since } \beta \ll 1$$

$$G(j\omega) \approx \frac{\frac{S_0}{1-k_0}}{j\omega + \frac{1}{l}(1-k_0)}.$$

For the purpose of the asymptotic approximation the format of the transfer function should be changed by division by  $\frac{1-k_0}{l}$  to give

$$G(j\omega) = \frac{\frac{S_0 l}{(1-k_0)^2}}{(\frac{l}{1-k_0})j\omega + 1}.$$

The calculations were carried out by using this equation and the resulting "Bode" plots are shown in Figs. 10 and 11 for the amplitude in db.

From the Bode plots the values of the transfer function at 10 cps were obtained and compared with the experimentally determined transfer function in Fig. 13.



## RESULTS

## Discussion

The results are shown in tabular form in Table 1 and are plotted in Figs. 12 and 13 for transfer function phase angle and amplitude, respectively, as functions of the multiplication factor. No attempt was made to draw a curve for the phase angle. The fact that a given phase angle may result from more than one value of the multiplication factor makes shutdown measurement by means of that quantity impossible.

The amplitude of the transfer function appears to be a smooth function of the multiplication factor, with some deviation from the curve for the two points representing the condition closest to critical. The rate of increase in amplitude for the two points is less than would be expected both based on the experimental and on the theoretical curves. However, at this point the count rate became too high for the  $\text{BF}_3$  detector, and the voltage from the count rate circuit decreased. It is apparent that if one detector is to be used for the entire range from critical down to a maximum negative reactivity, an ionization chamber in connection with a  $\mu\text{m}$  ammeter would be more suitable than a  $\text{BF}_3$  detector and a count rate circuit.

If an envelope were drawn in Fig. 13 to include the 68% confidence level within which the transfer functions were

determined, and neglecting the point with the largest deviation, the multiplication factor may be found in this envelope with a confidence of  $\pm 0.04\%$  near critical and with a confidence of  $\pm 0.17\%$  in the fully shutdown condition. For the point omitted above it is found that any given determination of the transfer function can result in an uncertainty of  $\pm 0.33\%$ .

Since the transfer function is the measured quantity it is fortunate that the signal to noise ratio allows it to be determined with the highest level of confidence in the steepest portion of the curve.

#### Sources of Error

Errors in any type of measurement are unavoidable, but whether these errors result in a range of uncertainty around some average value of the measurement, or whether they will be systematic, causing the measurement to deviate from some theoretically true value as a function of some of the input variables, will depend on the nature of the errors. The random errors can be minimized by proper experimental procedures and improved techniques and equipment; the systematic errors may be eliminated by introducing compensation by changing the method of experimentation or by computational procedures applied after the experiment.

The experiment involved several components, each of which

was a potential error source. These components, their errors and how they are accounted for will be outlined.

The transfer function input was a sinusoidal variation in the multiplication factor which was induced by an oscillator, driven by a D.C. motor. The motor speed was adjusted to 10 cps at the beginning of the experiment and it was not necessary to make further adjustments. The tachometer indicating the speed was continuously connected with the motor and was checked periodically throughout any given run. Although any deviation in speed would have been easily detected, it may be assumed that it could have varied undetected within the least estimable division on the tachometer. The tachometer was set to read 600 rpm; the least division was 20 rpm and a tenth of a division could be estimated. For variation within this range, the motor could vary within  $600 \pm 2$  rpm or  $10 \pm 0.033$  cps. This will cause a maximum deviation to occur in the transfer function at the region near critical because there the transfer function varies as 20 db/decade at 10 cps (Fig. 10). For the maximum transfer function the theoretical amplitude was 34 db at 10 cps; with the uncertainty in speed postulated the amplitude varies between 34.07 db and 33.93 db which corresponds to an uncertainty of app.  $\pm 0.8\%$ . The maximum deviation in the transfer function due to variation in motor speed was thus 0.8%.

The minimum deviation in the transfer function will occur at the large negative reactivity where the 10 cps line intersects a flat portion of the curve (Fig. 11). This, however, is where the largest deviation was found in the measurement, and it is therefore believed that variation in the measured quantity due to variation in motor speed was insignificant at any reactivity. It is also apparent that any such error is random in nature and will appear as an uncertainty in the final answer.

Another source of error which may have been present in the input was the synchronization between the oscillator and the sine-generating potentiometer. The two were being driven by the same shaft (Fig. 5) and the sine wave induced in the flux was synchronized with the sine wave from the potentiometer as explained earlier. After the two were synchronized the assembly was removed from the reactor and the alignment was checked visually with a protractor. The center of the oscillator could be found within a range of approximately two degrees. If such an error had been present, it would have had the effect of shifting the measured phase by the same amount. The effect on the amplitude would be to change it by an amount depending on the total phase angle, thus the amplitude could change by 1.4% near critical where the measured phase angle was  $69.28^\circ$  and by 8.1% at the shut down position where the measured phase angle was  $25^\circ$ .

The electronic equipment which measured and amplified the output from the reactor consisted of many components and each of these components is limited in accuracy within a manufactured tolerance. By associating a transfer function with each stage through which the signal passes from its detection until it is multiplied by the signal from the sine pot, it is apparent that, depending on these transfer functions, the signal may undergo a phase shift and an amplification through each stage. If the frequency of the oscillator had not been constant it would have been necessary to know the exact transfer function of the equipment since the phase shift and amplification may be frequency dependent. This knowledge is unnecessary when the frequency is constant and when it is not necessary to know the exact value of the reactor transfer function.

The random error which may be introduced by the electronic equipment is due to voltage fluctuations in the detector high voltage supply and the various electronic noises mentioned earlier. With the cross-correlation method these will be integrated out over an infinite integration period, and random errors present in the final answer will be due to short integration periods. These periods were limited by the maximum voltage capacity of the analog computer and the necessity for having the signals into the multipliers as close as possible to the maximum.

In order to check the cross-correlation method for its ability to eliminate noise, an uncorrelated signal, obtained by placing the detector near a neutron source, was cross-correlated with the signal from the sine pot. Four runs were made, each of approximately 50 second duration, cross-correlating the noise signal with both the sine and the cosine signals. These signals averaged about zero which showed that the cross-correlation procedure worked.

The procedure was also checked by cross-correlating the sine signal with the cosine signal, resulting in an output signal of zero on one of the integrators. This was the expected result and gave a check on the equipment.

The final source of error in the experiment came from the method of reading the cross-correlation function. For this purpose a two-channel chart recorder was used, in conjunction with a voltmeter located on the analog computer. The recorder was operated at a speed depending on the magnitude of the signal. If the signal increased rapidly as function of time the recorder was operated at a speed of 5 mm per second and if the signal increased slowly the recorder was operated at 1 mm per second. The recorder speed was checked against a watch at both speeds over a period of 30 seconds and no deviation was found.

In reading the values on the chart the measurement of time was done at a graduation to eliminate estimation. The

voltage was then read at the same graduation and could be estimated to 0.1 volt.

Errors introduced in this manner were of the random type since they were the result of estimates in reading. The recordings were carried out until one of the channels reached 10 volts as indicated on the voltmeter and the effect on that channel of having an uncertainty of  $\pm 0.1$  volt would be  $\pm 1\%$ . The minimum reading on the other channel was on the order of 3 volts which with the same reading accuracy would result in an uncertainty of  $\pm 3.3\%$ . When the result is a product of two variables the relative uncertainty of the result is the sum of the relative uncertainties of the components (3). The uncertainty in establishing one value of the transfer function is then a maximum of  $\pm 4.3\%$ . The uncertainty here is in the form of a variation, so that the variation of the mean will be  $1/n$  times the variation of the individual. The minimum number of observations taken was 6, so that the maximum uncertainty arising from the reading of the graph was  $\pm 0.7\%$ .

The total random variation which can be related to the equipment and the observational methods are due to the possible variation in motor speed and the uncertainty of reading the graphs. The maximum random variation from these sources is  $\pm 1.5\%$ .

Reproducibility of the experiment was checked by

making measurements with a different  $\text{BF}_3$  detector of a slightly smaller volume and located at the same position in the reactor. The measurements were made for a range of reactivities from almost critical down to maximum negative reactivity. Apparently the second detector was more efficient than the first, because in spite of its smaller volume the output from the count rate circuit was higher. The normalization of the second set of data to the first was done at the beginning of the experiment by adjusting the gain through the amplification stage such that the output when viewed on the oscilloscope was the same as for the first detector when the reactor was in the same state. The data from the second detector fell on the same smooth curve as the initial data.

#### Comparison with Theoretical

The purpose of this investigation was to find a quick and relatively simple method for measuring the shutdown margin of the UTR-10 reactor, and agreement with a theoretical model was not considered important. The theoretical expression for the transfer function was useful for determining the possibility of the method and for indicating the range of values to be expected from the measurements. The UTR-10 reactor consists of two coupled cores but was approximated by point reactor kinetics in the development of the



theoretical expression. It is believed that this approximation accounts for the major difference in the experimental and theoretical curves shown in Fig. 13.

The difference may be explained by the fact that whereas the reactivity transfer function depends on the whole of the reactor, the measured transfer function was obtained at one point in the reactor. The two would be different particularly in this case where the oscillator was placed in the region between the two coupled cores, and the detector was placed against the outside of one of the cores.

In general the transfer function is spatially dependent which points out the necessity of locating the oscillator and the detector in the same positions for every experiment in order to obtain reproducible data.

## SUMMARY AND CONCLUSIONS

A scheme was devised which measured the shutdown margin of the UTR-10 reactor. The measurement was accomplished by finding the reactor transfer function at a frequency of 10 cps and relating that quantity to the multiplication factor. The transfer function is a measure of the response of the reactor to some disturbance. In this experiment the disturbance was in the form of a sinusoidal variation in localized reactivity, induced by an oscillator. The output was in the form of the measured flux.

Measurements made with two different detectors fell on the same curve of transfer function vs. reactivity and with the same level of uncertainty, indicating that the reproducibility is good.

In order to use the procedure for determining the shutdown margin of another reactor it is necessary to establish a calibration curve for that reactor. This may be done by the use of calibrated control rods or insertion of calibrated absorbers. As the reactor burnup increases there will be a slight change in  $\beta$ , the delayed neutron fraction, and  $l$ , the neutron lifetime. These changes will result in a change of the transfer function. However, the effect on the transfer function will be to change the break frequency, and since 10 cps occurs past the break frequency at almost all of the reactivities, the measured transfer function curve should

only change slightly up or down while keeping its original shape. An occasional calibration could be performed with the reactor critical at some low power level, by either using the adjustable gain to arrive at the same measured transfer function, or by using a normalization factor.

## RECOMMENDATIONS FOR FURTHER STUDY

Several possibilities exist for improving the accuracy of the measurements and the ease with which the transfer function is obtained.

As was mentioned earlier, the use of an ion chamber instead of a  $\text{BF}_3$  detector would extend the usable range of the instrument, because an ionization chamber can be used over at least a  $10^6$  - fold range of neutron flux (15). The ion chamber will also have less noise associated with it because it is less sensitive to background  $\gamma$  and  $\alpha$  radiation. Since the noise is the greatest single contributor to uncertainty its partial elimination is bound to increase the confidence level.

Another possible improvement in confidence level would be to change from a D.C. motor to a synchronous motor to eliminate uncertainty about the oscillator frequency.

In order to facilitate handling of the data, the use of the analog computer may be extended to give the amplitude of the transfer function directly. This will allow a much faster determination of that function. Because of the shorter time involved, more data could be collected and a better confidence level established.

The transfer function could also be obtained directly by the use of a servo motor geared to the sine pot on the oscillator. If a position read-out were provided for the

sine pot, its position could be changed so that the phase lag of the transfer function were zero, and the angle through which it moved could be read. The cross-correlation would then yield the amplitude of the transfer function directly.

It should be interesting to investigate the possibility of correlating the experimental with the theoretical transfer functions for a subcritical reactor. This would make it possible to arrive at a better theoretical description of a coupled core reactor.

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